Skewness in Stock Returns, Periodic Cash Payouts, and Investor Heterogeneity*

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Abstract

This paper analyzes the asset pricing implications of periodic cash payouts within the context of a stationary rational expectations model with heterogeneous investors. The periodicity of cash payouts provides a natural motivation for time-varying conditional volatility in stock returns. I show that the unconditional distribution of returns is a mixture of normals distribution, which has non-trivial skewness properties. I examine how conditional volatility, trading volume and skewness in stock returns are related to information dispersion and liquidity in the stock market. The model provides a rationale for negative skewness in aggregate stock returns–while generating positive skewness in firm level returns–which is based on cross-sectional dispersion of event dates. Preliminary evidence on this prediction is given.

Key words: Skewness, investor heterogeneity, period cash payouts, turnover

JEL Classifications: G12, G14, D82

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1 Introduction

Aggregate stock market returns display negative skewness–a propensity to generate negative returns with greater probability than suggested by the normal distribution function.[1] A large body of literature has been devoted to explaining this stylized fact about the distribution of aggregate stock returns (e.g., Black, 1976, Christie, 1982, Blanchard and Watson, 1982, Pindyck 1984, French et al., 1987, and Hong and Stein, 2003). The evidence on aggregate returns contrasts with the other stylized fact that firm-level returns are positively skewed. In this paper, I provide a unified theory for both stylized facts and give evidence consistent with the theory.

This paper develops a stationary, heterogeneous investor, asset pricing model of firm-level events. Specifically, I model periodic cash payouts.[2] The periodicity of cash payouts motivates conditional heteroskedasticity in stock returns. With periodic payouts, cash flow news is discounted according to the time left till the next payout. Thus, the impact of news on the conditional return volatility is greater for news released closer to the payout. This gives rise to a pattern of increasing conditional return volatility, despite constant unconditional volatility of news. In addition, discounting also implies that the conditional return volatility increases at an increasing rate. The presence of a risk-return trade off in the model implies that these properties apply equally to expected returns and induce positive skewness in expected returns.

Investors are heterogeneous with respect to their information sets and their investment opportunity sets. Following Wang (1994), informed investors receive private information signals about the stock and can trade a private investment opportunity besides the publicly traded stock. The return to the private investment is positively correlated with the stock return giving rise to rebalancing trades. These rebalancing trades mask the informed trades by the same informed investors and prevent the equilibrium from being fully revealing.

Both trading motives–rebalancing trading and informed trading–induce informed investors to buy as the payout date approaches. When buying, informed investors bid up the stock price ahead of the payout to induce the uninformed investors to sell. The price increase partially offsets the steep increase in expected returns that otherwise prevails because of the periodicity in payouts. Both rebalancing or liquidity shocks and information asymmetry work

[1] See, for example, Kon (1984), Chen et al. (2001), and Bris et al. (2007).

[2] Cash payouts is a catch all for dividends, share buybacks, and share issuances. For brevity and notational simplicity, below I shall use cash payout and dividends interchangeably.

to flatten expected returns closer to payouts and reduce skewness in expected returns.

I show that the equilibrium unconditional distribution of stock returns is a mixture of normals distribution. Under a mixture of normals distribution, skewness in stock returns is given by two components. One is skewness in expected returns. The other captures the association between expected returns and conditional return variance. Numerical exercises of comparative statics suggest that skewness in expected returns can be an important determinant of skewness in stock returns. Therefore, while the assumption of periodic dividends tends to generate positive skewness in stock returns, liquidity shocks and information asymmetry tend to reduce this skewness.

To explain the apparent disconnect between firm-level return skewness and market return skewness, I offer a composition result due to cross-sectional variation in cash payout dates. I make two complementary hypotheses. First, negative skewness in large portfolios is a composition result. Second, this composition result is explained by cross-sectional variation in cash payout dates. Before describing the results note that skewness measured on a portfolio return is the mean skewness of the firms in the portfolio plus co-skewness terms that describe how one firm's return comoves with another firms squared deviation from the mean.[3]

Consider a stock market composed of i.i.d. copies of the single firm that I study, where firms differ only in their cash payout date. When firms have different cash payout dates, the high return volatility of some firms around their payout date contrasts with the low volatility of other firms' returns at the same date. This generates negative co-skewness in return volatility but also negative co-skewness in expected returns due to the presence of a risk-return trade-off in the model. If stocks in a portfolio display negative co-skewness with each other, then the portfolio may have negative skewness even if each of the firm returns has positive skewness: The fact that some stocks have low returns while another stock has high volatility (and high expected returns) may mean that the *portfolio* composed of the two firms has a somewhat larger probability of low outcomes than predicted by the normal distribution.

I provide preliminary evidence consistent with both hypotheses. First, I show evidence that negative skewness is an composition result. By construction, sample skewness on a portfolio of stocks is the sum of mean firm-level skewness of the firms in the portfolio and co-skewness terms. If firm-level skewness is positive on average, negative portfolio skewness

[3]This decomposition is similar to that obtained for the variance of a portfolio return which equals mean variance of the firms in the portfolio plus covariance terms.

must come from co-skewness and be a property derived from the aggregation of stocks. To show this, I compute the mean and median skewness over six months of CRSP daily return data in successively larger equally-weighted portfolios. The data shows a systematic negative relationship between number of firms in a portfolio and skewness: Mean and median skewness decrease monotonically as the number of firms in a portfolio increases.

To produce a more direct test of the model, note that the critical feature in generating negative co-skewness in the model is the cross-sectional dispersion of event dates and the corresponding cross-sectional dispersion of stock return volatility. The finding of a monotonic relation between skewness and portfolio size is thus consistent with the second model prediction in that it is likelier that dispersion in volatility across-firms increases with the number of firms in a portfolio.

In addition, I use information on the cross-sectional dispersion of earnings announcements which have been shown to also be associated with brief periods of high volatility (Ball and Khotari, 1991). I compare the within-industry spread between mean firm-level skewness and industry-wide skewness across industries with varying degrees of cross-sectional variation in earnings announcement dates. This spread is driven only by the co-skewness terms and, all else equal, the model predicts that it should be larger in industries with greater cross-sectional variation in announcement dates. The evidence suggests that, indeed, industries with greater dispersion of earnings announcement dates have larger spreads and, moreover, a larger fraction of semesters with negative skewness in aggregate returns.

The other study that I am aware that tries to explain the apparent disconnect between firm-level and aggregate skewness in stock returns is Duffee (1995). Duffee tests the hypothesis that aggregate stock returns have negative skewness due to a common factor that is negatively skewed. He finds that idiosyncratic firm returns are positively skewed and that a common factor is also positively skewed.

The model is also consistent with other stylized findings in the data. First, while mean and median firm-level return skewness are positive in the data, a significant fraction of firms display negative skewness. As discussed above, the model in this paper is able to generate negative skewness when asymmetric information and liquidity trading are important providing new insights relative to existing models that also predict negative skewness. Many studies have focused on asymmetric volatility as an explanation for negative skewness: Black (1976) and Christie (1982) with the leverage effect; Pindyck (1984), French et al. (1987), Campbell and Hentschel (1992), and Veronesi (2004), with the volatility feedback effect; Bekaert and Wu (2000) and Wu (2001); Blanchard and Watson (1982) because of stock price bubbles bursting; and, Hong and Stein (2003) because short sales constraints limit the market's ability to incorporate bad news.[4]

Second, Chen et al. (2001) documents that skewness in firm-level returns is negatively associated with own-firm turnover. In my model, the effect of liquidity shocks on turnover is endogenous and depends on the strength of the rebalancing trades. There is a direct effect by which larger liquidity shocks generate more rebalancing trades and increase turnover. There are also two indirect effects of larger liquidity shocks. One is to lower the conditional correlation between the stock return and the return to the private investment opportunity, which then reduces the amount of rebalancing trades and turnover. The other is to reduce uninformed investors' adverse selection problem, which increases turnover. For small liquidity shocks, turnover increases with larger liquidity shocks. At the same time, larger liquidity shocks reduce skewness which is consistent with the evidence. Hong and Stein (2003) also predict that skewness is negatively related to turnover, but Bris et al. (2007) do not reject the hypothesis that firm-level skewness is unrelated to the presence of short sales restrictions (likewise, see Bae et al., 2006).

Third, the model in this paper is also consistent with facts on dividend and earnings announcements. Kalay and Loewenstein (1985) show that dividend announcements are associated with high returns and high volatility of stock returns. Ball and Khotari (1991) show that the high expected returns around earnings announcements are also associated with high volatility. In addition, informed investors tend to buy before earnings announcements and to sell after earnings announcements: Kaniel et al. (2008) show that individuals intense buying prior to earnings announcements is associated with positive abnormal returns following the announcement and post-announcement sales by individuals and attribute about 50% of their finding to private information trading.

Fourth, there is evidence that firm-level stock returns are well described by a mixture of normals distribution (Kon, 1984, Zangari, 1996, and Haas et al., 2004).

The model is related to the literature that analyzes the flow of information and its implications for trading behavior (e.g., He and Wang, 1995), and the literature that studies

[4]There is also a large literature that documents that skewness is priced, be it total skewness (e.g. Arditti, 1967), co-skewness (e.g., Kraus and Litzenberger, 1976, and Harvey and Siddique, 2000), or idiosyncratic skewness (e.g., Boyer, et al., 2009). By working with portfolio returns, these models appear to capture well the fact that portfolio returns are generally negatively skewed. Bakshi et al. (2003) and Conrad et al. (2009) study ex ante skewness in firm and portfolio returns.

trading volume around public news events in models with information asymmetries (e.g., Kim and Verrecchia, 1991, 1994, and Kandel and Pearson, 1995). This paper complements that literature by providing a stationary asset pricing model of event studies and by focusing on a different set of moments on returns and trading. Albuquerque and Miao (2009) study a framework where investors also have private information about future dividends that is uncorrelated with current dividends. They use that setting to study momentum and reversals in stock returns.

The paper is organized as follows. The next section describes the model and section 3 describes the equilibrium properties of the model. Section 4 analyzes the effect of periodic cash payouts, liquidity and information asymmetry. Section 5 discusses the sources of skewness in the model and section 6 studies the relation between skewness, information asymmetry and liquidity trading. Section 7 presents preliminary evidence on the composition hypothesis and section 8 concludes. The appendix contains the proofs of the propositions in the main text and some additional results.

2 The Model

The model economy is composed of infinitely lived investors with differential information about the underlying value of the stock. Investors also differ in their access to investment opportunities. The model is further detailed next.

2.1 Investment opportunities

Investors trade in a riskless asset which has a perfectly elastic supply at the gross rate of return $R > 1$. There is a single stock in fixed supply of 1. Each share of the stock is infinitely divisible and trades competitively in the stock market at any time t at the ex-dividend price P_t. The stock pays a dividend every $K + 1$ periods,

$$D_t = F_t + \sum_{j=0}^{K} \varepsilon_{t-K+j}^{D}.$$

It is understood that if t corresponds to a period $k > 0$, then $D_t = 0$.[5] The dividend can be decomposed into a persistent component,

$$F_t = \rho_F F_{t-1} + \varepsilon_t^F, \qquad 0 \leq \rho_F \leq 1,$$

[5]The assumption of non-stochastic periodicity in cash payouts is made for simplicity. If the timing of cash payouts is stochastic, it is conceivable that it would constitute another source of conditional heteroskedasticity in stock returns.

with $\varepsilon_t^F \sim N\left(0, \sigma_F^2\right)$, and a transitory component, $\sum_{j=0}^{K} \varepsilon_{t-K+j}^D$. Information about the transitory component is revealed every period in the form of shocks $\varepsilon_t^D \sim N\left(0, \sigma_D^2\right)$.

Trading periods are further identified by a superscript $k = 0, ..., K$, where $k = 0$ refers to a dividend-paying period and $k = 1, ..., K$ refer to non-dividend-paying periods. Below, I often identify a period with (t, k). I then write the excess return in a dividend-paying period as

$$Q_t^0 \equiv P_t^0 + D_t - RP_{t-1}^K,$$

and in a non-dividend-paying period as

$$Q_t^k \equiv P_t^k - RP_{t-1}^{k-1}.$$

Every period, investors get two public signals about the next dividend,

$$\begin{aligned} S_t^{Fk} &= F_t + \varepsilon_t^{Fk}, \\ S_t^{Dk} &= \varepsilon_t^D + \varepsilon_t^{Dk}, \end{aligned}$$

where $\varepsilon_t^{Fk} \sim N\left(0, \sigma_{Fk}^2\right)$ and $\varepsilon_t^{Dk} \sim N\left(0, \sigma_{Dk}^2\right)$. The quality of the public information determines the level of asymmetric information. When information is infinitely precise and $\sigma_{Dk}^2 = \sigma_{Fk}^2 = 0$, there is no asymmetric information across investors.

Lastly, as in Wang (1994), there is another risky asset, available only to informed investors in the spirit of Merton (1987), which pays the excess return,

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q, \tag{1}$$

at time $t + 1$. The excess return in this private investment opportunity is composed of a persistent expected excess return,

$$Z_t = \rho_Z Z_{t-1} + \varepsilon_t^Z, \qquad 0 \leq \rho_Z \leq 1,$$

with $\varepsilon_t^Z \sim N\left(0, \sigma_Z^2\right)$ and a transitory unexpected return $\varepsilon_t^q \sim N\left(0, \sigma_q^2\right)$. It is assumed that $E\left(\varepsilon_t^D \varepsilon_t^q\right) = \sigma_{Dq} > 0$. Except for the correlation between ε_t^D and ε_t^q no other correlation between shocks exists.

It is important to note that the shocks described above are all conditionally homoskedastic.[6] Therefore, any conditional heteroskedasticity in equilibrium prices and returns is endogenously generated. If, in contrast, I were to allow the shocks described above to display conditional heteroskedasticity, then the conditional heteroskedasticity in equilibrium prices and returns would follow trivially.

[6] I allow heteroskedasticity in the public information signals for generality, but the numerical examples below assume homoskedastic public information.

2.2 Investors' problem

There is a continuum of identical, informed investors denoted by the superscript i, and a continuum of identical, uninformed investors denoted by the superscript u. The mass of informed investors is λ and the mass of uninformed investors is $1-\lambda$. Investors choose their time t asset allocation to maximize utility over next period wealth,

$$-E\left[\exp^{-\gamma W_{t+1}}|\mathcal{I}_t^j\right].$$

For informed investors, $j=i$, the maximization is over the holdings of the riskless asset, of the stock, θ_t, and of the private investment opportunity, α_t, and is subject to the budget constraint,

$$W_{t+1}=Q_{t+1}^{k+1}\theta_t+\alpha_t q_{t+1}+RW_t,$$

and the information set,

$$\mathcal{I}_t^i=\left\{P_{t-s},D_{t-s},S_{t-s}^F,S_{t-s}^D,F_{t-s},Z_{t-s},\varepsilon_{t-s}^D\right\}_{s\geq 0}.$$

Uninformed investors, $j=u$, face a similar budget constraint, but with $\alpha_t\equiv 0$, and have the information set,

$$\mathcal{I}_t^u=\left\{P_{t-s},D_{t-s},S_{t-s}^F,S_{t-s}^D\right\}_{s\geq 0}.$$

For any variable x_t, its conditional expectation under $\mathcal{I}_t^u$ is denoted by $\hat{x}_t=E^u\left[x_t|\mathcal{I}_t^u\right]=E_t^u\left[x_t\right]$.

2.3 Definition of equilibrium

Investors trade the stock competitively in the stock market, making their asset allocation while taking prices as given. In equilibrium, the stock price is such that the market for the stock clears:

$$\lambda\theta_t^i+(1-\lambda)\,\theta_t^u=1. \tag{2}$$

3 Stock Market Equilibrium

3.1 The equilibrium state vector and price function

In solving for the stock market equilibrium, I start with a guess for the state vector; the vector that contains the information needed to price the stock. In principle, the full history of shocks may be relevant, but because of the information hierarchy present in the information sets of the two investor groups, the equilibrium may be described via a finite-dimensional state vector. To further understand the components of the state vector, consider for example the information relevant at time $(t, 0)$, a dividend-paying period. Because dividends have just been paid, the information contained in $\sum_{j=0}^{K} \varepsilon_{t-K+j}^{D}$ is not useful to forecast the next dividend payment and information about F_t and Z_t suffices. Next, consider the period immediately after a dividend-paying period. Because investors now have information about F_t, Z_t, and ε_t^D, and ε_t^D is to be paid later, then all three are needed. In keeping with this reasoning, I guess the state vector to be

$$\mathbf{x}_t = \left[F_t, Z_t, \varepsilon_t^D, ..., \varepsilon_{t-K}^D\right]^\intercal .$$

The state vector has a fixed dimension of $K+3$. As suggested above, at times this state vector may contain too much information, but this formulation guarantees stationarity and is easier to work with. Letting $\boldsymbol{\varepsilon}_t^k = \left[\varepsilon_t^F, \varepsilon_t^Z, \varepsilon_t^D, \varepsilon_t^q, \varepsilon_t^{Fk}, \varepsilon_t^{Dk}\right]^\intercal$, with $\boldsymbol{\Sigma}_{\varepsilon\varepsilon}^k = E\left[\boldsymbol{\varepsilon}_t^k \boldsymbol{\varepsilon}_t^{k\intercal}\right]$, the appendix shows that the dynamics of the state vector can be represented via the constant matrices $\mathbf{A}_x$ and $\mathbf{B}_x$:

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t^k. \tag{3}$$

Note that I allow the vector of residuals to depend on k as the noise in the public information may vary across periods.

It is natural to guess the time (t, k) stock price –corresponding to k periods after the last dividend payment– to depend on both the state vector $\mathbf{x}_t$ and the uninformed investors' expectation of the state vector, $\hat{\mathbf{x}}_t = E_t^u[\mathbf{x}_t]$:

$$P_t^k = p^k + \mathbf{p}_i^k \mathbf{x}_t + \mathbf{p}_u^k \hat{\mathbf{x}}_t. \tag{4}$$

The next proposition further characterizes the equilibrium price function.

Proposition 1 *In equilibrium, the price function is given by,*

$$\begin{aligned} P_t^k &= p^k + \frac{(\rho_F/R)^{K+1-k}}{1-(\rho_F/R)^{K+1}} F_t + R^{-(K+1-k)} \sum_{j=0}^{k-1} \varepsilon_{t-j}^D \qquad (5) \\ &\quad + p_{i2}^k Z_t - p_{u1}^k \left(F_t - \hat{F}_t\right) - \sum_{j=0}^{k-1} p_{u3+j}^k \left(\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D\right), \end{aligned}$$

for any $k = 0, ..., K$. The price function is such that $p^k < 0$, and $p_{i2}^k < 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$.

The first line of the price function describes the present value of dividends when all information is public. The present value calculation accounts for the fact that at time (t,k), it takes $K+1-k$ periods until dividends are paid. The transitory shock ε^D enters the stock price function because (some) investors learn about it before it is paid as dividend. The price coefficients on ε^D shocks reflect the necessary discounting: ε_t^D enters the price function at time t with a coefficient of $R^{-(K-k)}$, whereas ε_{t+1}^D enters the price function at time $t+1$ with a coefficient of $R^{-(K-k-1)} > R^{-(K-k)}$. Despite being transitory, ε_t^D has de facto persistence of one until the next dividend payment and persistence of zero thereafter.

The coefficient on F_t is more complex. At a dividend paying period, the stock resembles a perpetuity discounted at rate $(R/\rho_F)^{K+1} - 1$ because F_t dies out over time according to ρ_F but continues to matter for dividends even after the next dividend payment. The analogy of a perpetuity also helps explain why the effect of discounting is magnified for small values of the interest rate or large values of the persistence coefficient ρ_F:

$$\frac{(\rho_F/R)^{K+1-k}}{1-(\rho_F/R)^{K+1}} \to \infty \text{ as } \rho_F/R \to 1.$$

The second line has two components. The first is the effect of rebalancing by informed investors: When the two assets are positively correlated an increase in the expected return of the private investment opportunity Z_t shifts some of the weight that informed investors put on the stock to the private investment, thus lowering the stock price. The second component describes the effect of asymmetric information. With $p_{uj\geq 0}^k$, knowing that the stock is better than uninformed investors think it is, i.e., $F_t - \hat{F}_t > 0$ or $\varepsilon_{t+j}^D - \hat{\varepsilon}_{t+j}^D > 0$, gives informed investors knowledge that the price is lower than it should be if it is to reflect the present discounted value of dividends.

Finally, note that the price function does not depend on $\hat{Z}_t$. The reason for this is that uninformed investors' forecasting errors are correlated implying that one of the elements of $\hat{\mathbf{x}}_t$ is not priced. Formally, uninformed investors learn $\Pi^k \equiv \mathbf{p}_i^k \mathbf{x}_t$ from observing prices, which implies

$$\mathbf{p}_i^k \mathbf{x}_t = E_t^u \left(\mathbf{p}_i^k \mathbf{x}_t \right) = \mathbf{p}_i^k \hat{\mathbf{x}}_t. \tag{6}$$

3.2 Uninformed investors' filtering problem

Over time, the quantity of information available to uninformed investors changes, namely to reflect the fact that when $k=0$ dividends are paid and may be used to forecast the ability to pay dividends in the future. Specifically, at every non-dividend-paying period

(t,k), uninformed investors observe the vector $\mathbf{y}_t^k = \left[\Pi_t^k, S_t^{Fk}, S_t^{Dk}\right]^\intercal$, whereas at dividend-paying periods $(t,0)$, uninformed investors observe $\mathbf{y}_t^0 = \left[\Pi_t^0, S_t^{F0}, S_t^{D0}, D_t\right]^\intercal$. The appendix shows that it is possible to write the vector of observables making use of the time-varying matrices $\mathbf{A}_y^k$ and $\mathbf{B}_y^k$,

$$\mathbf{y}_t^k = \mathbf{A}_y^k \mathbf{x}_t + \mathbf{B}_y^k \boldsymbol{\varepsilon}_t^k, \qquad k = 0, ..., K.$$

Define the conditional volatilities:

$$\boldsymbol{\Sigma}_{xx}^k = \mathbf{B}_x \boldsymbol{\Sigma}_{\varepsilon\varepsilon}^k \mathbf{B}_x^\intercal,\ \boldsymbol{\Sigma}_{yy}^k = \mathbf{B}_y^k \boldsymbol{\Sigma}_{\varepsilon\varepsilon}^k \mathbf{B}_y^{k\intercal},$$

and $\boldsymbol{\Omega}_t = E_t^u\left[\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right)\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right)^\intercal\right]$. The next proposition gives the solution to uninformed investors' filtering problem.

Proposition 2 *The steady state Kalman filter is given by the $K+1$ matrices $\left\{\Omega^k\right\}_{k=0,...,K}$ that recursively solve the system of $K+1$ Riccati equations:*

$$\begin{aligned} \boldsymbol{\Omega}^k &= \left(\mathbf{I} - \mathbf{K}_k^{k-1}\mathbf{A}_y^k\right)\left(\mathbf{A}_x \Omega^{k-1}\mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}^k\right) \\ \mathbf{K}_k^{k-1} &= \left(\mathbf{A}_x \Omega^{k-1}\mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}^k\right)\mathbf{A}_y^{k\intercal}\left(\mathbf{A}_y^k\left(\mathbf{A}_x \Omega^{k-1}\mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}^k\right)\mathbf{A}_y^{k\intercal} + \boldsymbol{\Sigma}_{yy}^k\right)^{-1}. \end{aligned}$$

If t is a dividend paying period and $k=0$, it should be understood that $k-1$ stands for K. Uniformed investors forecast of the state vector is given by

$$\hat{\mathbf{x}}_t = \mathbf{A}_x \hat{\mathbf{x}}_{t-1} + \mathbf{K}_k^{k-1}\hat{\boldsymbol{\varepsilon}}_t^k, \tag{7}$$

and

$$\mathbf{y}_t^k = \mathbf{A}_y^k \mathbf{A}_x \hat{\mathbf{x}}_{t-1} + \hat{\boldsymbol{\varepsilon}}_t^k. \tag{8}$$

The residual $\hat{\boldsymbol{\varepsilon}}_t^k = \mathbf{y}_t^k - E_{t-1}^u\left[\mathbf{y}_t^k\right]$ is normally distributed with mean zero and covariance matrix $Var_{t-1}^u\left(\hat{\boldsymbol{\varepsilon}}_t^k\right)$ given in the appendix.

I show below that the modeling feature of periodic dividends, which guarantees that the observation vector is time varying, is sufficient to generate conditional heteroskedasticity in returns even in the absence of asymmetric information. An additional source of time variation in stock return volatility is induced by a changing correlation between the stock return and the private investment return, $\rho_{Qq,k}^i$. A third source of time-varying volatility in stock returns results from conditional variation in Ω^k due to endogenous changes in the level of asymmetric information.

3.3 Optimal asset allocation

From the optimization problem of both investors, I derive the optimal holdings for the stock for each investor,

$$\theta_t^i = \frac{E_t^i\left[Q_{t+1}^{k+1}\right]}{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} - \frac{\rho_{Qq,k}^i E_t^i\left[q_{t+1}\right]}{\gamma\sigma_{Q,k}^i\sigma_q^i\left(1-\left(\rho_{Qq,k}^i\right)^2\right)}, \tag{9}$$

$$\theta_t^u = \frac{E_t^u\left[Q_{t+1}^{k+1}\right]}{\gamma\left(\sigma_{Q,k}^u\right)^2}. \tag{10}$$

Demand for the stock is composed of a myopic term for both investors. In addition, informed investors may trade the stock to hedge the risk in other components of their portfolio. The hedging demand term reflects the fact that when the stock is positively correlated with the private investment, i.e., $\rho_{Qq}^i > 0$, buying more of both assets increases portfolio risk and is undesirable.

To construct the demands, I calculate the expected return of the various assets under the different information sets. Using equation (5), the appendix shows that

$$E_t^i\left[Q_{t+1}^{k+1}\right] = e_0^{k+1} + e_{i2}^{k+1}Z_t - e_{u1}^{k+1}\left(F_t - \hat{F}_t\right) - \sum_{j=0}^{k-1} e_{uj+3}^{k+1}\left(\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D\right), \tag{11}$$

and

$$E_t^u\left[Q_{t+1}^{k+1}\right] = e_0^{k+1} + e_{i2}^{k+1}\hat{Z}_t. \tag{12}$$

Finally, the expected return for the private investment opportunity is $E_t^i\left[q_{t+1}\right] = Z_t$. Substituting these expressions in the asset demands gives the next result.

Proposition 3 *The equilibrium stock demand functions are*

$$\theta_t^i = f_{i0}^{k+1} + f_{i1}^{k+1}Z_t + f_{i2}^{k+1}\left(F_t - \hat{F}_t\right) + \sum_{j=0}^{k-1} f_{uj+3}^{k+1}\left(\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D\right), \tag{13}$$

$$\theta_t^u = f_{u0}^{k+1} + f_{u1}^{k+1}\hat{Z}_t, \tag{14}$$

where $f_{i0}^{k+1}, f_{u0}^{k+1} > 0$, *and* $f_{u1}^{k+1} > 0$ *and* $f_{i1}^{k+1} < 0$ *if and only if* $Cov_t^i\left(Q_{t+1}, q_{t+1}\right) > 0$.

These stock demands guarantee stock market clearing. In equilibrium, uninformed investors trade only to accommodate what they think are rebalancing trades by informed investors as dictated by movements in $\hat{Z}_t$: Uninformed investors buy when they perceive that informed investors are selling for liquidity reasons (i.e., $\hat{Z}_t$ is large), thus expecting a positive return (see equation (12)). Informed investors trade the stock to rebalance their portfolio – by reducing their holdings when private opportunities abound – and to make use of their private information – by increasing their holdings when uninformed investors underestimate either F_t or ε_{t-j}^D. As can be seen from equation (11) and the fact that $e_{i2}^{k+1} > 0$, informed investors sell the stock in response to a rebalancing shock, notwithstanding the high expected returns. The loss of high future returns on the stock sold comes at the benefit of hedging some of the risk from taking a larger position in the private investment opportunity.

4 Properties of Stock Return and Trading Volume

This section analysis the dependence of the distribution of stock returns and trading activity on k, the parameter which identifies the distance from the next dividend-paying period.

4.1 The role of periodicity in dividends

To isolate the role of the assumption of periodic dividends, consider a benchmark economy with a representative investor who trades only the stock and has information $\mathcal{I}_t^i$. In this economy the equilibrium is characterized by a price function identical to that in equation (5) with $p_{i2}^k = p_{uj}^k = 0$ (see the appendix). Further, the following are moments of the stock return conditional on k alone,

$$E_k\left[Q_{t+1}^{k+1}\right] = \gamma Var_k\left(Q_{t+1}^{k+1}\right), \tag{15}$$

with

$$Var_k\left(Q_{t+1}^{k+1}\right) = \left(\frac{(\rho_F/R)^{K+1-k}}{1-(\rho_F/R)^{K+1}}\right)^2 \sigma_F^2 + R^{-2(K-k)}\sigma_D^2. \tag{16}$$

The main result from this benchmark model states that conditional stock return volatility increases with k despite the fact that the shocks ε_t^F and ε_t^D have constant unconditional volatility of σ_F^2 and σ_D^2, respectively. Intuitively, ε_t^F and ε_t^D are only paid as dividends in period $t+K+1-k$, and their impact on the stock price at t reflects the necessary discounting.[7] This means that the impact of news on stock return volatility is also discounted, but less so as k increases. In addition, it is easy to show that discounting also implies that the conditional volatility is convex in k.

[7] As argued before the effect of discounting can be made infinitely large by having $\rho_F/R \to 1$.

In this benchmark model of a representative investor with myopic asset demand, equation (15) states that the expected stock return is proportional to the conditional stock return volatility. Therefore, expected returns increase monotonically, and are convex in k. These properties have implications for the distribution of expected returns. In particular, lower values of expected returns occur for smaller k and are relatively closer together than higher values of conditional returns, which tend to occur more spaced out. The intuition is that news that occur long before dividend payments occur are highly discounted and contribute little to risk, giving rise to periods of concentrated low expected returns. News that occur close to dividend payments impact prices more and contribute more to risk, giving rise to peaks of volatility around dividend announcements when expected returns are also high. This asymmetry in expected returns means that the distribution of expected returns is positively skewed and, as I will show below, is an important determinant of skewness in stock returns.

4.2 The added effect from rebalancing trades

In this subsection, I extent the benchmark model above to allow for heterogeneity in investors' investment opportunity sets, while maintaining the assumption of identical information sets.[8] In the presence of other correlated assets (see (1)), informed investors trade to rebalance their portfolio giving rise to non-trivial volume properties. The appendix shows that the equilibrium is described by a price function identical to that in equation (5) with $p_{uj}^{k} = 0$ and $p_{i2}^{k} < 0$ for all k, from which I can derive the equilibrium conditional mean returns and variance:

$$E_k\left[Q_{t+1}^{k+1}\right] = \gamma Var_k\left(Q_{t+1}^{k+1}\right)\frac{1-\rho_{Qq,k}^2}{\lambda+(1-\lambda)\left(1-\rho_{Qq,k}^2\right)}, \tag{17}$$

with

$$Var_k\left(Q_{t+1}^{k+1}\right) = \left(\frac{(\rho_F/R)^{K+1-k}}{1-(\rho_F/R)^{K+1}}\right)^2\sigma_F^2 + R^{-2(K-k)}\sigma_D^2 + \left(p_{i2}^{k+1}\right)^2\sigma_Z^2. \tag{18}$$

The conditional covariance of the stock return with the private investment return is

$$Cov_{Qq,k} = R^{-(K-k)}\sigma_{Dq}. \tag{19}$$

Combining (10) with (17), I obtain,

$$E_k\left[\theta_t^u\right] = \frac{1-\rho_{Qq,k}^2}{\lambda+(1-\lambda)\left(1-\rho_{Qq,k}^2\right)}. \tag{20}$$

[8] For consistency, I maintain the label "informed" investors even though in this version of the model all investors share the same information set.

Investors shift their asset holdings to reflect their subjective risk exposures. Specifically, a high correlation $\rho^2_{Qq,k}$ means that informed investors can hedge more of the risk associated with the stock and wish to hold more of it.

The presence of the private investment opportunity produces the following three new effects on conditional moments. First, the returns of the stock and private investment are correlated and their covariance increases over time (see equation (19)). The intuition for the increasing covariance is that at time (t, k) the contribution of the risky dividend cash flow to the stock price is affected by discounting, $R^{-(K-k)}\varepsilon^D_{t+1}$, limiting the amount of hedging that can be done with the private investment return. Second, holding fixed the volatility of returns, expected returns decrease relative to (15) provided $\rho^i_{Qq,k} \neq 0$. Intuitively, a fraction λ of the investors now perceives the stock to be less risky and only price the risk that cannot be hedged with the private investment opportunity. Third, the conditional stock return variance increases relative to (16) by the term $\left(p^k_{i2}\right)^2 \sigma^2_Z$. This term is due to liquidity shocks that affect the stock price via informed investors' portfolio rebalancing.

To proceed, I resort to numerical methods because the model does not lend itself to a complete analytical solution. The parameters chosen do not represent a proper calibration of the model but illustrate qualitative patterns that are found to be robust. I focus on the moments discussed above and also on the conditional mean trading volume, $E_k\left[Vol_t\right]$. Trading volume is defined as $Vol_t = (1-\lambda)\left|\theta^u_t - \theta^u_{t-1}\right| = (1-\lambda)\left|\Delta\theta^u_t\right|$, and its conditional mean is

$$E_k\left[Vol_t\right] = (1-\lambda)\sqrt{\frac{2}{\pi}}\sigma_{\Delta\theta^u_t,k}.$$

Mean trading volume is derived from the fact that volume has a Chi distribution with one degree of freedom. Mean trading volume fluctuates with the volatility of uninformed investors' net acquisitions, $\sigma^2_{\Delta\theta^u_t,k}$.

Figure 1 illustrates the equilibrium properties under two scenarios, low versus high σ^2_Z. For sufficiently low values of σ^2_Z, the rebalancing effect is of second order for the variance of returns and the last term in (18) is small. It helps to consider the limiting case of $\sigma^2_Z \to 0$. The conditional return volatility equals that in the benchmark model. It increases with k and moreover increases with k faster than the conditional covariance. Thus, the correlation of the stock return with the private investment return declines monotonically with k. As k increases, the stock becomes an increasingly poor hedging asset for informed investors' private investment, and their stock holdings decrease with k (see equation (20)). To encourage uninformed investors to buy at an increasing rate there must be an increasing dispersion in expected returns at high values (even relative to the dispersion in conditional variance values), generating positive skewness in expected returns over and above the skewness predicted by the periodicity of dividends.

When $\sigma_Z^2 > 0$, the effect described above is complemented with a liquidity effect acting via the conditional volatility of returns. As σ_Z^2 increases, the liquidity effect becomes the main driver of the conditional variance of returns because the weight of the term $\left(p_{i2}^{k+1}\right)^2 \sigma_Z^2$ on total variance increases. In particular, I find numerically that the new variance term is relatively flat with respect to k compared to the other variance terms.[9] This has two effects. First, it leads to a less skewed distribution of the conditional return variance and, by (17), a less skewed distribution of expected stock returns. Second, it leads to an increasing conditional correlation $\rho_{Qq,k}^2$,[10] thus increasing informed investors' willingness to buy more of the stock on average as k increases. In equilibrium, uninformed investors, which provide the needed liquidity, are compensated by selling at high prices before the dividend announcement. Both effects suggest a flattening of expected returns prior to the dividend announcement and lower skewness in expected returns (see Figure 1). The presence of liquidity shocks thus offsets the increase in expected returns prior to dividend announcements caused by the periodicity of dividends.

Rebalancing trades lead to non-trivial trading volume as seen in Figure 1. The main difference between the low and high σ_Z^2 cases regarding trading volume is that in the former, conditional trading volume is decreasing in k, whereas in the later it is increasing in k. This difference is caused by the patterns of the conditional correlation of stock returns and private investment returns. When the conditional correlation increases with k, informed investors' hedging demand introduces increasing variation in stock holdings in response to the volatility in the expected private investment return, i.e., σ_Z^2.

4.3 The complete model

In the complete model, investors differ also in their information sets, $\mathcal{I}_t^i$ and $\mathcal{I}_t^u$. Information asymmetry introduces a third effect into conditional moments. Namely, despite some revelation of information via the price, informed investors may accumulate private information

[9] For sufficiently small σ_Z^2, the liquidity effect may increase the skewness in $\sigma_{Q,k}^2$, thus increasing skewness in conditional expected returns.

[10] To summarize the monotonicity properties of $\rho_{Qq,k}^2$ with respect to k, it decreases with k for low σ_Z^2, but it increases in k for sufficiently large σ_Z^2.

as k increases thus reducing the risk they face when holding the stock and increasing their expected return.

With asymmetric information the expected return becomes (see the appendix),

$$E_k\left(Q_{t+1}^{k+1}\right) = \frac{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)\left(\sigma_{Q,k}^u\right)^2}{\lambda\left(\sigma_{Q,k}^u\right)^2+(1-\lambda)\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)}. \tag{21}$$

As before, the expected return increases with risk aversion and with increases in the risk of the stock as perceived by either informed or uninformed investors. Combining (10) with (21), I obtain the conditional mean holdings of uninformed investors,

$$E_k\left[\theta_t^u\right] = \frac{\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)}{\lambda\left(\sigma_{Q,k}^u\right)^2+(1-\lambda)\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)}. \tag{22}$$

In an equilibrium where informed investors accumulate private information as k increases, their conditional return variance decreases with k relative to that of uninformed investors and their conditional mean holdings increase.

It is not possible to obtain a (quasi-) closed form solution to the conditional variance of stock returns, $\sigma_{Q,k}^2 = Var_k\left(Q_{t+1}^{k+1}\right)$. The conditional variance differs from those in the previous models in that asymmetric information changes the way shocks affect prices and also introduces additional volatility via the forecast errors of uninformed investors.

To analyze the properties of the model, I evaluate the equilibrium numerically in the case of maximal asymmetry of information, $\sigma_{Fk}^2, \sigma_{Dk}^2 \to \infty$. Figure 2 presents the equilibria that result under two scenarios, small and large σ_Z^2.[11] The two cases produce similar, qualitative implications for the conditional return and variance and for conditional mean holdings of uninformed investors. These implications are also quite similar to those in Figure 1 for the case of large σ_Z^2. The presence of asymmetric information generates an incentive for informed investors to buy the stock as the date of the dividend payment nears. This is because the information asymmetry is highest just before the dividend is paid, even after accounting for information revealed through the market price. The stock price rises to induce uninformed investors to sell well before dividend payments, which further flattens and possibly induces a hump shape in expected stock returns. Skewness in expected returns decreases and may even become negative.

[11] In Figure 2, I use a lower value for σ_Z^2 than in Figure 1 since otherwise the juxtaposition of the plots would not allow for a clear observation of the various patterns.

What makes the case of low σ_Z^2 stand out is its implications for mean trading volume. When σ_Z^2 is low, information trades tend to dominate rebalancing trades worsening the adverse selection problem. As informed investors' private information accumulates with k, the adverse selection problem becomes more acute, and trading volume dries up prior to dividend announcements. This pattern is consistent with evidence in Lee et al. (1993) who show that spreads widen and depths fall in anticipation of earnings announcements.

5 Equilibrium Skewness

5.1 Unconditional distribution of returns

Conditionally, equilibrium excess stock returns are normally distributed. Unconditionally, however, they are not normal because the mean and variance of a randomly drawn return observation depend on k. Hence, because a k-period stock return is drawn from a normal density $\phi\left(Q; e_0^k, \sigma_{Q,k}^2\right)$ and such observations occur with frequency $1/\left(K+1\right)$, the unconditional distribution of returns is a mixture of normals distribution. Formally,

Proposition 4 *The unconditional distribution of stock returns is a mixture of normals distribution with density*

$$f\left(Q\right) = \frac{1}{K+1}\sum_{k=0}^{K}\phi\left(Q; e_0^k, \sigma_{Q,k}^2\right),$$

where $\phi\left(.\right)$ is the normal density function.

The periodicity of dividends, by generating time-varying conditional volatility in stock returns, leads to the derived mixture of normals distribution for stock returns for $K \geq 1$. This result provides a theoretical justification for attempting to fit a mixture of normals distribution for stock returns (e.g., Kon, 1984).

In the appendix, I prove the following corollary.

Corollary 1 *The unconditional mean and variance of stock returns are*

$$\begin{aligned} E\left(Q_{t+1}\right) &= \frac{1}{K+1}\sum_{k=0}^{K} e_0^k, \\ Var\left(Q_{t+1}\right) &= \frac{1}{K+1}\sum_{k=0}^{K}\left[\sigma_{Q,k}^2 + \left(e_0^k - E\left(Q_{t+1}\right)\right)^2\right]. \end{aligned}$$

The unconditional skewness in stock returns is

$$E\left[(Q-E(Q_{t+1}))^3\right]=\frac{1}{K+1}\sum_{k=0}^{K}\left(e_0^k-E(Q_{t+1})\right)^3+\frac{3}{(K+1)^2}\sum_{k=0}^{K}\sum_{j<k}\left(\sigma_{Q,k}^2-\sigma_{Q,j}^2\right)\left(e_0^k-e_0^j\right). \tag{23}$$

The unconditional mean return is simply the mean of the k-conditional expected returns, $e_0^k=E_k\left(Q_{t+1}^{k+1}\right)$. The unconditional mean variance is an average of the k-conditional variances plus the mean of squared deviations of each of the k-conditional means to the unconditional mean.

5.2 Skewness in firm level returns

Skewness in stock returns can be broken down into two parts. The first term in (23) is the level of skewness in expected returns, e_0^k. This term is positive (negative) when a small number of expected returns display high (small) values. The second term describes the impact on skewness of the comovement between return volatility with expected returns. Loosely speaking, this second term is related to the leverage and feedback hypotheses that entertain a negative correlation between current low *realized* returns (hence, high *expected* returns) and high conditional volatility.

When $K=0$, returns are unconditionally normally distributed and skewness is zero. When $K=1$, because each k-period is weighted equally, it is straightforward to show that $\sum_{k=0}^{1}\left(e_0^k-E(Q_{t+1})\right)^3=0$. Skewness then becomes

$$E\left[(Q-E(Q_{t+1}))^3\right]=\frac{3}{4}\left(\sigma_{Q,0}^2-\sigma_{Q,1}^2\right)\left(e_0^0-e_0^1\right). \tag{24}$$

Positive skewness is thus an implication of a risk-return trade off; when periods of high expected returns are associated with periods of high volatility. Furthermore, the stronger the risk-return trade off, the higher is the skewness. When $K>1$, the first term in (23) is no longer necessarily zero and skewness in expected returns also matters. Thus, unconditional skewness in returns becomes harder to sign. However, it remains that the second term in (23) is positive when $\sigma_{Q,k}^2>\sigma_{Q,j}^2$ and $e_0^k>e_0^j$. Therefore, while a risk-return trade off tends to generate positive skewness, it is no longer a necessary nor a sufficient condition.

5.3 Skewness in aggregate returns

This subsection considers two hypotheses to explain the negative skewness in aggregate returns. First, that a composition effect explains the *negative* skewness in marketwide returns when firm level returns are *positively* skewed. Second, that this composition effect arises from cross-sectional heterogeneity in cash payout dates. To isolate this composition effect, consider a portfolio with firms that are i.i.d. copies of the firm I study above. Together with the assumptions of negative exponential utility and normality of shocks, the assumption of i.i.d. copies, guarantees that the optimal investor decisions and equilibrium outcomes for each individual stock are unchanged. In particular, these assumptions guarantee that stock returns are uncorrelated with one another, so that the wedge between marketwide return skewness and firm level skewness is not caused by such correlations. This is a very stark construction meant to isolate the composition effect of heterogeneity in cash payout dates. It is reasonable to think, for example, that the number private investment opportunities is smaller then the number of stocks traded and that several stocks may serve as a hedge to the same private investment opportunity. Such setting would allow for stock returns to be correlated in equilibrium, but would potentially introduce confounding effects when demonstrating that the composition effect *alone* can drive a wedge between firm level skewness and marketwide skewness.

The composition effect can best be described by considering a stock market with two firms, labelled 1 and 2, each with one share. The stock market dollar return is thus $Q_{Mt} = Q_{1t} + Q_{2t}$ and its unconditional distribution is

$$f(Q_M) = \frac{1}{K+1}\sum_{k=0}^{K}\phi\left(Q_M; e_{01}^k + e_{02}^k, \sigma_{Q1,k}^2 + \sigma_{Q2,k}^2\right),$$

where, for simplicity, I abuse notation slightly and have the index k for firm 2 refer to the time after the last cash payout for firm 1. Unconditional skewness is

$$\begin{aligned} E\left[(Q_{Mt} - E(Q_{Mt}))^3\right] &= S_1 + S_2 + \frac{3}{K+1}\sum_{k=0}^{K}\left(e_{01}^k - E(Q_1)\right)^2\left(e_{02}^k - E(Q_2)\right) \\ &\quad + \frac{3}{K+1}\sum_{k=0}^{K}\left(e_{01}^k - E(Q_1)\right)\left(e_{02}^k - E(Q_2)\right)^2 \\ &\quad + \frac{3}{K+1}\sum_{k=0}^{K}\left(\left(e_{01}^k - E(Q_1)\right)\sigma_{Q2,k}^2 + \left(e_{02}^k - E(Q_2)\right)\sigma_{Q1,k}^2\right), \end{aligned}$$

where S_i is the skewness of firm i as in Corollary 1. Market skewness is given by the sum of firm skewness in stock returns and co-skewness in expected returns across all firms. Adding more firms implies that after considering all the skewness and co-skewness values (with different weights), one has to also consider the product, $\Pi_i\left(e_{0i}^k - E(Q_i)\right)$, of every combination of three firms' returns.

Negative co-skewness can bring the value of market skewness below that of firm skewness. This is achieved by having volatile periods for firm 1–with mean returns above the unconditional mean–coincide with less volatile periods for firm 2–with mean returns below the unconditional mean–and vice-versa. This way, the portfolio may end up with a low return more often than predicted by a normal distribution. Figure 3 illustrates how a market portfolio can have negative co-skewness. Figure 3 uses the same parameters as in Figure 2, but with more informative signals, $\sigma_{Dk}^2 = \sigma_{Fk}^2 = 0.5$, so that firm skewness is positive. In both panels of Figure 3, firm level return skewness is given by the horizontal line parallel to the x-axis. In panel A, I plot the equilibrium market skewness when the market is composed of two firms with cash payout dates at periods 0 and k, respectively, where $k = 0, ..., K$. The maximum effect of co-skewness is produced when announcements are farthest apart in time. In panel B, I plot the equilibrium market skewness when the stock market is composed of $k+1$ firm types with cash payout dates at periods $0, 1, ..., k$, respectively. Moving to the right along the x-axis represents an increase in the number of firms in the stock market. Co-skewness is negative and can be so large that market skewness becomes negative even when firm level skewness is positive. The negative co-skewness results from the cross-sectional dispersion in return volatility.

6 Liquidity, Asymmetric Information, and Skewness

6.1 Liquidity shocks and skewness

Consider the model of subsection 4.2, now to analyze the effects of liquidity shocks on skewness. Recall that an increase in σ_Z^2 tends to generate lower and possibly negative skewness in expected returns, which contributes to lower skewness in stock returns. In addition, liquidity shocks also impact the risk-return trade off. The main channel is through a decrease in the skewness of conditional volatility of returns for large σ_Z^2, which decreases the risk return trade off. For low σ_Z^2, liquidity shocks may amplify the dispersion in the conditional volatility, leading to greater skewness in stock returns. Figure 4 suggests that by and large the former two effects dominate, inducing a negative association between liquidity shocks and skewness in stock returns.

The monotonicity of unconditional mean volume, $(K+1)^{-1} \sum_k E_k [Vol_t]$, with respect to σ_Z^2, depends on the relative strength of two factors. First, σ_Z^2 has a direct positive effect on the volatility of holdings through the volatility of the conditional private investment return.

This is the traditional effect of liquidity shocks. Second, σ_Z^2 has an indirect negative effect through ρ_{Qq} which declines as σ_Z^2 increases, discouraging overall rebalancing by informed investors. Figure 4 shows that the former dominates for low values of σ_Z^2, whereas the later effect dominates for high values of σ_Z^2.

Variation in the relative size of liquidity shocks can thus produce a negative association between skewness and turnover at low values of σ_Z^2, while generating positive skewness in returns. In this range, liquidity shocks increase turnover. At the same time, liquidity shocks decrease the skewness in expected returns because informed investors become increasingly eager to hold relatively more of the stock close to a dividend announcement and encourage uninformed to sell (or to buy less) by bidding up the price prior to the dividend announcement and flattening subsequent expected returns. This prediction is consistent with the evidence in Chen et al. (2001) that high turnover is associated with lower skewness.

6.2 Asymmetric information and skewness

Consider now the complete model where agents are heterogeneous also with respect to their information sets. I start by first illustrating the effects of liquidity on skewness in this model, and then move on to discuss the effects of information asymmetry.

Figure 5 plots skewness and turnover for various levels of the liquidity shock σ_Z^2 assuming maximal asymmetry of information, $\sigma_{Fk}^2, \sigma_{Dk}^2 \to \infty$. As σ_Z^2 increases, informed investors are better able to mask their information trades and less information is revealed through the price. This gives informed investors an extra incentive to hold more of the stock as the dividend payment approaches. Hence, they must bid the stock price up long before that in order to buy from uninformed investors. This trading pattern generates a hump-shape in expected returns which translates into negative skewness in expected stock returns. Because skewness in expected returns is a major source of variation in skewness, stock returns also become negatively skewness for large σ_Z^2.

Turnover is increasing in σ_Z^2 over a wider range of values for σ_Z^2 relative to the model without asymmetric information. The reason is that in the presence of asymmetric information, larger liquidity shocks contribute to more turnover by decreasing uninformed investors' adverse selection problem. Thus, the model with asymmetric information also predicts a negative association between turnover and skewness.

The effect of asymmetric information on skewness and turnover is depicted in Figure 6. In the figure, I allow σ_{Dk}^2 and σ_{Fk}^2 to vary from 0.01 (low information asymmetry) to 0.96 (high information asymmetry). The increase in information asymmetry, for fixed σ_Z^2, leads to lower skewness as suggested above. For turnover, the increase in information asymmetry produces a non-monotonic pattern. A negative association is to be expected when the adverse selection effect is strong enough. A positive association may arise if the level of liquidity shocks is sufficiently large. To see this note that when $\sigma_{Dk}^2 = \sigma_{Fk}^2 \to 0$ only rebalancing trades exist. For small σ_{Dk}^2 and σ_{Fk}^2, trading volume may increase with noise in public news because informed investors can and will exploit their information advantage by trading on it besides trading for rebalancing reasons.

The model with asymmetric information predicts a negative association between skewness and volume caused by changes in the level of information asymmetry. In addition to predicting this negative relation and positive firm-level skewness, the model provides an additional testable hypothesis that can be used to distinguish it from Hong and Stein's (2003). In my model, the negative association between skewness and turnover is predicted only for low levels of information asymmetry (when the association is driven by changes in σ_{Fk}^2 and σ_{Dk}^2). Instead, in Hong and Stein, it requires large differences in opinions. It is up to further testing to determine how investor "disagreement" impacts the association between skewness and turnover.

7 Some Evidence on the Composition Effect

This section provides evidence consistent with the two hypotheses presented in subsection 5.3. The first hypothesis is that the finding of negative skewness in aggregate stock returns is a composition result. I construct equally weighted portfolios of $N = 1, 5, 25, 125, 625, Market$ firms using CRSP daily total return data from 1/1/1980 to 12/31/2008. Firms that do not have complete return data in each semester are dropped. The larger portfolio, labelled $Market$ comprises all of the firms in CRSP in a specific semester with complete return data. For $N = 1$, mean and median skewness equal mean and median firm level skewness. For $N > 1$, the portfolios are constructed in the following way. First, I assign a random number to each firm and rank firms accordingly. Second, non-overlapping portfolios are formed by taking each consecutive group of N firms according to their ranking. This procedure guarantees that if two firms are in the same portfolio for $N = 5$ they are also in the same portfolio for any $N > 5$–a property that is needed to capture the effect of increasing N. For example, when $N = 25$, each portfolio is composed of five portfolios with five firms each as constructed for the exercise with $N = 5$. Finally, mean and median skewness are then computed across all N-firm portfolios. The procedure is then repeated for every semester.

Table 1 and Figure 7 present the results using median skewness. The results using mean skewness are similar. As the table and figure show, there is a systematic monotonic decreasing effect of N on skewness. For all semesters in the sample, median skewness is positive when $N = 1$, decreases in N, and for most semesters becomes negative for large N.

To interpret this result note that the sample (non-standardized) skewness when there are N firms in a portfolio is:

$$
\begin{aligned}
T^{-1}\sum_t (r_t - \bar{r}_t)^3 &= T^{-1}\sum_t \left(N^{-1}\sum_{i=1}^{N}(r_{it} - \bar{r}_{it})\right)^3 \\
&= N^{-3}\sum_{i=1}^{N} T^{-1}\sum_t (r_{it} - \bar{r}_{it})^3 + \text{co-skewness terms.} \qquad (25)
\end{aligned}
$$

The skewness measure I compute is the standardized skewness equal to $T^{-1}\sum_t (r_t - \bar{r}_t)^3 / [T^{-1}\sum_t (r_t - \bar{r}_t)^2]^{3/2}$. Therefore, N^{-3} also shows up in the denominator and it is safe to ignore its effect on standardized skewness. The first term in (25) is the mean of firm-level skewness and is positive and invariant with N. The second term in (25) contains the co-skewness terms, which decrease as N increases. This term describes the way returns of different firms relate to each other. For example, when $N = 2:$

$$
\text{co-skewness terms} = 3T^{-1}2^{-3}\sum_t \left((r_{1t} - \bar{r}_{1t})^2 (r_{2t} - \bar{r}_{2t}) + (r_{1t} - \bar{r}_{1t})(r_{2t} - \bar{r}_{2t})^2\right).
$$

Because negative skewness in aggregate returns must come from negative co-skewness, the evidence establishes that negative skewness at the aggregate is a composition effect. Moreover, because co-skewness depends on the cross-sectional variation of squared deviations of returns from its mean, the evidence is consistent with the hypothesis that cross-sectional variation in volatility is an important determinant of skewness. The results for mean skewness are virtually identical and available upon request. Duffee (1995) conjectures that the existence of a negatively skewed common factor in returns could deliver the same pattern in skewness in firm-level and aggregate returns. However, Duffee finds that idiosyncratic firm returns are positively skewed and that a common factor is also positively skewed.

Next, I turn to the second hypothesis formulated above that the composition effect is driven by cross-sectional variation in cash payout dates. The critical property of the model in generating negative co-skewness is the cross-sectional dispersion of event dates and the corresponding cross-sectional dispersion of stock return volatility. To produce a more direct test of the model, I use information on the cross-sectional dispersion of earnings announcements, which are periods associated with higher volatility (Ball and Khotari, 1991).

Nishi and Fried (2008) give the cross-sectional dispersion in fiscal year-ends–therefore also in earnings announcement events–across NAICS industries.[12] The industry with greatest concentration in fiscal year-ends, as measured by the Herfindahl index, is NAICS 22-Utilities closely followed by NAICS 52-Finance and Insurance. Because the number of firms in NAICS 22 is about 250 per semester, I instead use NAICS 52 which has about 1,500 firms per semester. The industry with lowest concentration in fiscal year-ends is NAICS 44-45-Retail Trade closely followed by NAICS 31–33- Manufacturing. Because the number of firms in NAICS 44-45 is only about 270 per semester, I instead use NAICS 31-33 which has close to 3,000 firms per semester. The Herfindahl index for NAICS 31-33 is 0.46 and for NAICS 52 is 0.78.

Figure 8 plots the within-industry spread between industry-wide skewness and the mean of firm-level skewness for NAICS 52 and NAICS 31-33. As equation (25) shows this spread is composed of the co-skewness terms only. The figure uses data only from 2002 onwards since CRSP only contains information on NAICS starting on August 2001. Because NAICS 31-33 has greater dispersion of earnings announcement events, the theory predicts that it should have a lower and more negative spread than NAICS 52. The evidence suggests that indeed industries with smaller concentration of earnings announcement dates have larger spreads, i.e., more negative co-skewness. Moreover, NAICS 31-33 also presents a smaller fraction of semesters with positive skewness in aggregate returns (two out of 14 as opposed to six out of 14 for NAICS 52). The results for median skewness are again virtually identical and available upon request.[13] As a robustness check, I sample 1,500 firms randomly from NAICS 31-33 so that the exercise is not affected by the number of firms in each sector. The results are unchanged and available upon request. While these results are supportive of the theory, the availability of more data in future tests would allow for a conditional analysis of the spread and the treatment of alternative explanations.

[12] An additional source of cross-sectional dispersion in volatility is the fact that even firms with identical fiscal-year end often announce quarterly earnings with weeks apart.

[13] The results using NAICS 22 and NAICS 44-45 are similar though not as strong. NAICS 22 has higher (i.e., more positive spread) for ten out of 14 semesters. The much smaller number of firms in either sector however suggests that the sample statistics I compute are less precise.

8 Conclusion

This paper analyzes the asset pricing implications of periodicity in cash payouts within the context of a stationary rational expectations model of firm events with heterogeneous investors. The paper establishes that periodicity in cash payouts gives rise to time-varying conditional volatility in stock returns and is an underlying source for positive skewness in the model. The periodicity of cash payouts predicts that the unconditional distribution of returns is a mixture of normals distribution.

I claim that the disconnect between firm-level return skewness and marketwide return skewness is a composition result due to cross-sectional variation in cash payout dates. I present preliminary evidence in favor of this rationale for negative skewness in aggregate returns. The paper also demonstrates the effects of liquidity shocks and information asymmetry on conditional moments and skewness. I show that skewness in expected returns may be an important driver of skewness in stock returns. I also show that both liquidity shocks and asymmetric information may cause a negative association between skewness in stock returns and turnover consistent with the evidence.

Future theoretical research should aim to understand related sources of time variation in conditional moments in stock returns, including the fact that announcements of payouts are made with a lag relative to the actual payout or that many firms do not regularly have cash payouts. More research is also needed to find out the sources of variation in co-skewness, including the effect of aggregate news events.

The results in this paper are informative to the literature on rare disasters which builds on the negative skewness of aggregate stock returns to explain the equity premium puzzle (e.g. Rietz, 1988, and Barro, 2006). More research is needed to understand how rare disasters in the stock market are related to the cross section of return volatility besides being related to infrequent changes in the pricing kernel. Future empirical research should also try to determine how the level of investor disagreement affects the negative association between firm-level skewness and stock turnover and further test the composition effect as a driver of negative aggregate skewness.

Appendix

A Proofs

I start by proving Proposition 2 and then turn to the proofs of propositions 1 and 3. The proofs of propositions 1-3 build on the general approach developed in Albuquerque and Miao (2009).

Proof of Proposition 2: The matrices that describe the state vector dynamics in equation (3) are given by

$$\mathbf{A}_x = \begin{bmatrix} \rho_F & 0 & ... & & & 0 \\ 0 & \rho_Z & 0 & ... & & 0 \\ 0 & 0 & 0 & ... & & 0 \\ 0 & 0 & 1 & 0 & ... & 0 \\ ... & ... & ... & & ... & 0 \\ 0 & 0 & ... & 0 & 1 & 0 \end{bmatrix}, \qquad \mathbf{B}_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & ... & 0 & & & \\ & 0 & ... & 0 & & \\ & & 0 & ... & 0 & 0 \end{bmatrix},$$

and the conditional covariance matrix of the vector of residuals when period t is k periods after the last dividend payment is,

$$\mathbf{\Sigma}_{\varepsilon\varepsilon}^k = E\left[\boldsymbol{\varepsilon}_t^k \boldsymbol{\varepsilon}_t^{k\intercal}\right] = \begin{bmatrix} \sigma_F^2 & 0 & 0 & 0 & 0 & 0 \\ & \sigma_{Z^D}^2 & 0 & 0 & 0 & 0 \\ & & \sigma_D^2 & \sigma_{Dq} & 0 & 0 \\ & & & \sigma_q^2 & 0 & 0 \\ & & & & \sigma_{Fk}^2 & 0 \\ & & & & & \sigma_{Dk}^2 \end{bmatrix}.$$

Uninformed investors observe the following signals at any time t:

$$\begin{aligned} \mathbf{y}_t^0 &= \begin{bmatrix} \Pi_t^0 & S_t^{F0} & S_t^{D0} & D_t \end{bmatrix}^\intercal = \mathbf{A}_y^0 \mathbf{x}_t + \mathbf{B}_y^0 \boldsymbol{\varepsilon}_t^0, \\ \mathbf{y}_t^k &= \begin{bmatrix} \Pi_t^k & S_t^{Fk} & S_t^{Dk} \end{bmatrix}^\intercal = \mathbf{A}_y^k \mathbf{x}_t + \mathbf{B}_y^k \boldsymbol{\varepsilon}_t^k, \qquad k = 1, ..., K, \end{aligned}$$

where

$$\mathbf{A}_y^0 = \begin{bmatrix} \mathbf{p}_i^0 \\ \mathbf{c}_1 \\ \mathbf{c}_3 \\ \mathbf{c}_{-2} \end{bmatrix}, \qquad \mathbf{A}_y^k = \begin{bmatrix} \mathbf{p}_i^k \\ \mathbf{c}_1 \\ \mathbf{c}_3 \end{bmatrix},$$

$$\mathbf{B}_y^0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{B}_y^k = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The row vectors $\mathbf{c}_l$ have zeros in all columns except at position l. The row vector $\mathbf{c}_{-2}$ has ones everywhere except at column 2.

Define the conditional volatilities:

$$\mathbf{\Sigma}_{xx}^k = \mathbf{B}_x \mathbf{\Sigma}_{\varepsilon\varepsilon}^k \mathbf{B}_x^\intercal, \; \mathbf{\Sigma}_{yy}^k = \mathbf{B}_y^k \mathbf{\Sigma}_{\varepsilon\varepsilon}^k \mathbf{B}_y^{k\intercal},$$

and $\mathbf{\Omega}_t^k = E_t^u \left[(\mathbf{x}_t - \hat{\mathbf{x}}_t) (\mathbf{x}_t - \hat{\mathbf{x}}_t)^\intercal \right]$. The steady state Kalman filter is given by the $K+1$ matrices (see Anderson and Moore, 1979)

$$\mathbf{\Omega}^0, ..., \mathbf{\Omega}^K$$

that solve the Riccati equations

$$\begin{aligned} \mathbf{\Omega}^k &= \mathbf{\Omega}_k^{k-1} - \mathbf{K}_k^{k-1} \mathbf{A}_y^k \mathbf{\Omega}_k^{k-1} \\ \mathbf{K}_k^{k-1} &= \mathbf{\Omega}_k^{k-1} \mathbf{A}_y^{k\intercal} \left(\mathbf{A}_y^k \mathbf{\Omega}_k^{k-1} \mathbf{A}_y^{k\intercal} + \mathbf{\Sigma}_{yy}^k \right)^{-1} \\ \mathbf{\Omega}_k^{k-1} &= \mathbf{A}_x \Omega^{k-1} \mathbf{A}_x^\intercal + \mathbf{\Sigma}_{xx}^k. \end{aligned} \tag{A.1}$$

If t is a dividend paying period, replace Ω_k^{k-1} by Ω_0^K and $\mathbf{K}_k^{k-1}$ by $\mathbf{K}_0^K$. I then obtain the steady-state filters as described in the proposition. The residual $\hat{\boldsymbol{\varepsilon}}_t^k = \mathbf{y}_t^k - E_{t-1}^u \left[\mathbf{y}_t^k \right]$ is normally distributed with mean of zero and conditional covariance matrix

$$Var_{t-1} \left(\hat{\boldsymbol{\varepsilon}}_t \right) = \mathbf{A}_y^k \mathbf{A}_x \mathbf{\Omega}^{k-1} \mathbf{A}_x^\intercal \mathbf{A}_y^{k\intercal} + \left(\mathbf{A}_y^k \mathbf{B}_x + \mathbf{B}_y^k \right) \mathbf{\Sigma}_{\varepsilon\varepsilon}^k \left(\mathbf{A}_y^k \mathbf{B}_x + \mathbf{B}_y^k \right)^\intercal. \tag{A.2}$$

■

Proof of Proposition 1: Using (6), one of the variables in $\hat{\mathbf{x}}_t$ can be expressed as a linear function of other variables and thus drops from the price function. I drop $\hat{Z}_t$ by noting that

$$\hat{Z}_t = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{x}_t - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}_t, \tag{A.3}$$

where p_{i2} is the second element of $\mathbf{p}_i$, and $\mathbf{I}_{-2}$ conforms with the state vector and denotes the matrix that is the same as the identity matrix, except that the $(2,2)$ element equals zero. Thus, I set $p_{u2} = 0$.

Using the guessed price function in (4) write the excess return at time $t+1$, assuming that $t+1$ is a non-dividend paying period (i.e., $K-1 \geq k \geq 0$ at time t), as

$$
\begin{aligned}
Q_{t+1}^{k+1} &= p^{k+1} + \mathbf{p}_i^{k+1}\mathbf{x}_{t+1} + \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\hat{\mathbf{x}}_{t+1} - R\left(p^k + \mathbf{p}_i^k\mathbf{x}_t + \mathbf{p}_u^k\mathbf{I}_{-2}\hat{\mathbf{x}}_t\right) \\
&= p^{k+1} - Rp^k + \left(\mathbf{p}_i^{k+1}\mathbf{A}_x - R\mathbf{p}_i^k\right)\mathbf{x}_t + \left(\mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{A}_x - R\mathbf{p}_u^k\mathbf{I}_{-2}\right)\hat{\mathbf{x}}_t \\
&\quad + \mathbf{p}_i^{k+1}\mathbf{B}_x\boldsymbol{\varepsilon}_{t+1}^{k+1} + \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{K}_{k+1}^k\hat{\boldsymbol{\varepsilon}}_{t+1}^{k+1} \\
&= p^{k+1} - Rp^k + \left(\mathbf{p}_i^{k+1}\mathbf{A}_x - R\mathbf{p}_i^k\right)\mathbf{x}_t + \left(\mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{A}_x - R\mathbf{p}_u^k\mathbf{I}_{-2}\right)\hat{\mathbf{x}}_t + \mathbf{p}_i^{k+1}\mathbf{B}_x\boldsymbol{\varepsilon}_{t+1}^{k+1} \\
&\quad + \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{K}_{k+1}^k\left(\mathbf{A}_y^{k+1}\mathbf{A}_x\left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}\right)(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1}\mathbf{B}_x + \mathbf{B}_y^{k+1}\right)\boldsymbol{\varepsilon}_{t+1}^{k+1}\right) \\
&= e_0^{k+1} + \mathbf{e}_i^{k+1}\mathbf{x}_t + \mathbf{e}_u^{k+1}\hat{\mathbf{x}}_t + \mathbf{b}_Q^{k+1}\boldsymbol{\varepsilon}_{t+1}^{k+1}
\end{aligned}
$$

where the constants

$$
\begin{aligned}
e_0^{k+1} &= p^{k+1} - Rp^k && \text{(A.4)} \\
\mathbf{e}_i^{k+1} &= \mathbf{p}_i^{k+1}\mathbf{A}_x - R\mathbf{p}_i^k + \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{K}_{k+1}^k\mathbf{A}_y^{k+1}\mathbf{A}_x\left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}\right) && \text{(A.5)} \\
\mathbf{e}_u^{k+1} &= \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{A}_x - R\mathbf{p}_u^k\mathbf{I}_{-2} - \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{K}_{k+1}^k\mathbf{A}_y^{k+1}\mathbf{A}_x\left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}\right) && \text{(A.6)} \\
\mathbf{b}_Q^{k+1} &= \mathbf{p}_i^{k+1}\mathbf{B}_x + \mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{K}_{k+1}^k\left(\mathbf{A}_y^{k+1}\mathbf{B}_x + \mathbf{B}_y^{k+1}\right). && \text{(A.7)}
\end{aligned}
$$

To arrive at the third equality above, I use the fact that

$$
\begin{aligned}
\hat{\boldsymbol{\varepsilon}}_{t+1}^{k+1} &= \mathbf{y}_{t+1}^{k+1} - \mathbf{A}_y^{k+1}\mathbf{A}_x\hat{\mathbf{x}}_t \\
&= \mathbf{A}_y^{k+1}\left(\mathbf{A}_x\mathbf{x}_t + \mathbf{B}_x\boldsymbol{\varepsilon}_{t+1}^{k+1}\right) + \mathbf{B}_y^{k+1}\boldsymbol{\varepsilon}_{t+1}^{k+1} - \mathbf{A}_y^{k+1}\mathbf{A}_x\hat{\mathbf{x}}_t \\
&= \mathbf{A}_y^{k+1}\mathbf{A}_x(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1}\mathbf{B}_x + \mathbf{B}_y^{k+1}\right)\boldsymbol{\varepsilon}_{t+1}^{k+1} \\
&= \mathbf{A}_y^{k+1}\mathbf{A}_x\left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}\right)(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \left(\mathbf{A}_y^{k+1}\mathbf{B}_x + \mathbf{B}_y^{k+1}\right)\boldsymbol{\varepsilon}_{t+1}^{k+1},
\end{aligned}
$$

where the last equality follows from (A.3),

$$
Z_t - \hat{Z}_t = \frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}(\hat{\mathbf{x}}_t - \mathbf{x}_t),
$$

and from

$$
\begin{aligned}
\mathbf{x}_t - \hat{\mathbf{x}}_t &= \mathbf{I}_{-2}(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{c}_2^\intercal\left(Z_t - \hat{Z}_t\right) \\
&= \mathbf{I}_{-2}(\mathbf{x}_t - \hat{\mathbf{x}}_t) - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}(\mathbf{x}_t - \hat{\mathbf{x}}_t) \\
&= \left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal\frac{1}{p_{i2}^k}\mathbf{p}_i^k\mathbf{I}_{-2}\right)(\mathbf{x}_t - \hat{\mathbf{x}}_t).
\end{aligned}
$$

Conditional expected returns equal

$$\begin{aligned}E_t^i\left[Q_{t+1}^{k+1}\right] &= e_0^{k+1}+\mathbf{e}_i^{k+1}\mathbf{x}_t+\mathbf{e}_u^{k+1}\hat{\mathbf{x}}_t \\ E_t^u\left[Q_{t+1}^{k+1}\right] &= e_0^{k+1}+\left(\mathbf{e}_i^{k+1}+\mathbf{e}_u^{k+1}\right)\hat{\mathbf{x}}_t.\end{aligned}$$

Use (A.3) to substitute out $\hat{Z}_t$ in $E_t^u\left[Q_{t+1}^{k+1}\right]$ (because likely $e_{i2}^{k+1}\neq 0$) and derive

$$\begin{aligned}E_t^u\left[Q_{t+1}^{k+1}\right] &= e_0^{k+1}+\mathbf{e}_i^{k+1}\left[\mathbf{I}_{-2}\hat{\mathbf{x}}_t+\mathbf{c}_2^{\intercal}\hat{Z}_t\right]+\mathbf{e}_u^{k+1}\hat{\mathbf{x}}_t \\ &= e_0^{k+1}+\mathbf{e}_i^{k+1}\left[\mathbf{I}_{-2}\hat{\mathbf{x}}_t+\mathbf{c}_2^{\intercal}Z_t-\frac{1}{p_{i2}^k}\mathbf{c}_2^{\intercal}\mathbf{p}_i^k\mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t-\mathbf{x}_t\right)\right]+\mathbf{e}_u^{k+1}\hat{\mathbf{x}}_t \\ &= e_0^{k+1}+\mathbf{e}_i^{k+1}\left(\mathbf{c}_2^{\intercal}\mathbf{c}_2+\frac{1}{p_{i2}^k}\mathbf{c}_2^{\intercal}\mathbf{p}_i^k\mathbf{I}_{-2}\right)\mathbf{x}_t+\left[\mathbf{e}_i^{k+1}\left(\mathbf{I}_{-2}-\frac{1}{p_{i2}^k}\mathbf{c}_2^{\intercal}\mathbf{p}_i^k\mathbf{I}_{-2}\right)+\mathbf{e}_u^{k+1}\right]\hat{\mathbf{x}}_t \\ &= e_0^{k+1}+\tilde{\mathbf{e}}_i^{k+1}\mathbf{x}_t+\tilde{\mathbf{e}}_u^{k+1}\hat{\mathbf{x}}_t.\end{aligned}$$

Likewise, I proceed in the same fashion to obtain the expression for excess returns when $t+1$ is a dividend-paying period:

$$\begin{aligned}Q_{t+1}^0 &= p^0+\mathbf{p}_i^0\mathbf{x}_{t+1}+\mathbf{p}_u^0\mathbf{I}_{-2}\hat{\mathbf{x}}_{t+1}+\mathbf{c}_{-2}\mathbf{x}_{t+1}-R\left(p^K+\mathbf{p}_i^K\mathbf{x}_t+\mathbf{p}_u^K\mathbf{I}_{-2}\hat{\mathbf{x}}_t\right) \\ &= p^0+\left(\mathbf{p}_i^0+\mathbf{c}_{-2}\right)\left(\mathbf{A}_x\mathbf{x}_t+\mathbf{B}_x\boldsymbol{\varepsilon}_{t+1}^0\right)+\mathbf{p}_u^0\mathbf{I}_{-2}\left(\mathbf{A}_x\hat{\mathbf{x}}_t+\mathbf{K}_0^K\hat{\boldsymbol{\varepsilon}}_{t+1}^0\right)-R\left(p^K+\mathbf{p}_i^K\mathbf{x}_t+\mathbf{p}_u^K\mathbf{I}_{-2}\hat{\mathbf{x}}_t\right) \\ &= p^0-Rp^K+\left(\left(\mathbf{p}_i^0+\mathbf{c}_{-2}\right)\mathbf{A}_x-R\mathbf{p}_i^K\right)\mathbf{x}_t+\left(\mathbf{p}_u^0\mathbf{I}_{-2}\mathbf{A}_x-R\mathbf{p}_u^K\mathbf{I}_{-2}\right)\hat{\mathbf{x}}_t+\left(\mathbf{p}_i^0+\mathbf{c}_{-2}\right)\mathbf{B}_x\boldsymbol{\varepsilon}_{t+1}^0 \\ &\quad+\mathbf{p}_u^0\mathbf{I}_{-2}\mathbf{K}_0^K\left(\mathbf{A}_y^0\mathbf{A}_x\left(\mathbf{I}_{-2}-\mathbf{c}_2^{\intercal}\frac{1}{p_{i2}^K}\mathbf{p}_i^K\mathbf{I}_{-2}\right)\left(\mathbf{x}_t-\hat{\mathbf{x}}_t\right)+\left(\mathbf{A}_y^0\mathbf{B}_x+\mathbf{B}_y^0\right)\boldsymbol{\varepsilon}_{t+1}^0\right) \\ &= e_0^0+\mathbf{e}_i^0\mathbf{x}_t+\mathbf{e}_u^0\hat{\mathbf{x}}_t+\mathbf{b}_Q^0\boldsymbol{\varepsilon}_{t+1}^0,\end{aligned}$$

with the constants e_0^0, $\mathbf{e}_i^0$, $\mathbf{e}_u^0$ and $\mathbf{b}_Q^0$ appropriately defined. Expressions for $E_t^i\left[Q_{t+1}^0\right]$ and $E_t^u\left[Q_{t+1}^0\right]$ are obtained as before.

The volatility of stock returns conditional on $\mathcal{I}_t^i$ is

$$\left(\sigma_{Q,k}^i\right)^2\equiv E_t^i\left[\left(Q_{t+1}^{k+1}-E_t^i\left[Q_{t+1}^{k+1}\right]\right)^2\right]=\mathbf{b}_Q^{k+1}\Sigma_{\varepsilon\varepsilon}^{k+1}\left(\mathbf{b}_Q^{k+1}\right)^{\intercal},$$

and the volatility of stock returns conditional on $\mathcal{I}_t^u$ is

$$\left(\sigma_{Q,k}^u\right)^2\equiv E_t^u\left[\left(Q_{t+1}^{k+1}-E_t^u\left[Q_{t+1}^{k+1}\right]\right)^2\right]=\mathbf{e}_i^{k+1}\Omega^k\left(\mathbf{e}_i^{k+1}\right)^{\intercal}+\mathbf{b}_Q^{k+1}\Sigma_{\varepsilon\varepsilon}^{k+1}\left(\mathbf{b}_Q^{k+1}\right)^{\intercal}.$$

The volatility of private investment returns conditional on $\mathcal{I}_t^i$ is

$$\left(\sigma_q^i\right)^2\equiv E_t^i\left[\left(q_{t+1}-E_t^i\left[q_{t+1}\right]\right)^2\right]=\sigma_q^2.$$

Finally, the covariance between stock and private investment returns conditional on $\mathcal{I}_t^i$ is

$$Cov_{Qq,k}^i \equiv E_t^i\left[\left(Q_{t+1}^{k+1} - E_t^i\left[Q_{t+1}^{k+1}\right]\right)\left(q_{t+1} - E_t^i\left[q_{t+1}\right]\right)\right] = \mathbf{b}_Q^{k+1}\Sigma_{\varepsilon\varepsilon}^{k+1}\mathbf{c}_4^\intercal,$$

where $\mathbf{c}_4^\intercal$ is a column vector of zeros with 1 in the fourth position. Define $\rho_{Qq,k}^i$ as the conditional correlation between stock and private investment returns conditional on $\mathcal{I}_t^i$.

It is now possible to solve for the stock demands and find the stock price that clears the market. From the investors problems, the optimal asset demands are as in (9) and (10). Inserting the asset demands into the stock market clearing condition yields

$$\lambda\frac{E_t^i\left[Q_{t+1}^{k+1}\right]}{\gamma\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)} - \lambda\frac{\rho_{Qq}^i E_t^i\left[q_{t+1}\right]}{\gamma\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)} + (1-\lambda)\frac{E_t^u\left[Q_{t+1}^{k+1}\right]}{\gamma\left(\sigma_Q^u\right)^2} = 1.$$

After replacing the conditional expectations with the expressions above, I get the following set of equilibrium conditions:

$$\lambda\frac{e_0^{k+1}}{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} + (1-\lambda)\frac{e_0^{k+1}}{\gamma\left(\sigma_{Q,k}^u\right)^2} = 1,$$

$$\frac{\lambda\mathbf{e}_i^{k+1}}{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} - \frac{\lambda\rho_{Qq,k}^i\mathbf{c}_2}{\gamma\sigma_{Q,k}^i\sigma_{q,k}^i\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} + \frac{(1-\lambda)\,\mathbf{e}_i^{k+1}\left(\mathbf{c}_2^\intercal\mathbf{c}_2 + \frac{1}{p_{i2}^k}\mathbf{c}_2^\intercal\mathbf{p}_i^k\mathbf{I}_{-2}\right)}{\gamma\left(\sigma_{Q,k}^u\right)^2} = 0, \tag{A.8}$$

$$\lambda\frac{\mathbf{e}_u^{k+1}}{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} + (1-\lambda)\frac{\mathbf{e}_i^{k+1}\left(\mathbf{I}_{-2} - \frac{1}{p_{i2}^k}\mathbf{c}_2^\intercal\mathbf{p}_i^k\mathbf{I}_{-2}\right) + \mathbf{e}_u^{k+1}}{\gamma\left(\sigma_{Q,k}^u\right)^2} = 0. \tag{A.9}$$

The first set of equations gives, for all k,

$$e_0^{k+1} = \frac{\gamma\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)\left(\sigma_{Q,k}^u\right)^2}{\lambda\left(\sigma_{Q,k}^u\right)^2 + (1-\lambda)\left(\sigma_{Q,k}^i\right)^2\left(1-\left(\rho_{Qq,k}^i\right)^2\right)} > 0. \tag{A.10}$$

Using

$$\mathbf{e}_i^{k+1}\left(\mathbf{c}_2^\intercal\mathbf{c}_2 + \frac{1}{p_{i2}^k}\mathbf{c}_2^\intercal\mathbf{p}_i^k\mathbf{I}_{-2}\right) = e_{i2}^{k+1}\left[\begin{array}{ccccc} \frac{p_{i1}^k}{p_{i2}^k} & 1 & \frac{p_{i3}^k}{p_{i2}^k} & \ldots & \frac{p_{i,K+3}^k}{p_{i2}^k}\end{array}\right],$$

in (A.8), gives

$$\begin{aligned}
\frac{\lambda}{\gamma}\frac{e_{i1}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}^{k+1}\frac{p_{i1}^k}{p_{i2}^k}}{\left(\sigma_Q^u\right)^2} &= 0,\\
\frac{\lambda}{\gamma}\left[\frac{e_{i2}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}-\frac{\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}\right]+\frac{1-\lambda}{\gamma}\frac{e_{i2}^{k+1}}{\left(\sigma_Q^u\right)^2} &= 0,\\
\frac{\lambda}{\gamma}\frac{e_{i3}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}^{k+1}\frac{p_{i3}^k}{p_{i2}^k}}{\left(\sigma_Q^u\right)^2} &= 0,\\
&...,
\end{aligned}$$

which can be solved for

$$e_{i1}^{k+1} = -\frac{p_{i1}^k}{p_{i2}^k}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \quad \text{(A.11)}$$

$$e_{i2}^{k+1} = \frac{\lambda\left(\sigma_Q^u\right)^2}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \quad \text{(A.12)}$$

$$e_{i3}^{k+1} = \frac{p_{i3}^k}{p_{i1}^k}e_{i1}^{k+1}, \quad ...$$

Turning now to (A.9), use

$$\mathbf{e}_i^{k+1}\left(\mathbf{I}_{-2}-\frac{1}{p_{i2}^k}\mathbf{c}_2^\intercal\mathbf{p}_i^k\mathbf{I}_{-2}\right)=\left[\begin{array}{cccc} e_{i1}^{k+1}-e_{i2}^{k+1}\frac{p_{i1}^k}{p_{i2}^k} & 0 & e_{i3}^{k+1}-e_{i2}^{k+1}\frac{p_{i3}^k}{p_{i2}^k} & ...\end{array}\right],$$

to derive

$$\begin{aligned}
0 &= \frac{\lambda}{\gamma}\frac{e_{u1}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}+(1-\lambda)\frac{e_{i1}^{k+1}-e_{i2}^{k+1}\frac{p_{i1}^k}{p_{i2}^k}+e_{u1}^{k+1}}{\gamma\left(\sigma_Q^u\right)^2},\\
0 &= \frac{\lambda}{\gamma}\frac{e_{u2}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}+(1-\lambda)\frac{e_{u2}^{k+1}}{\gamma\left(\sigma_Q^u\right)^2},\\
0 &= \frac{\lambda}{\gamma}\frac{e_{u3}^{k+1}}{\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}+(1-\lambda)\frac{e_{i3}^{k+1}-e_{i2}^{k+1}\frac{p_{i3}^k}{p_{i2}^k}+e_{u3}^{k+1}}{\gamma\left(\sigma_Q^u\right)^2},\\
& ...
\end{aligned}$$

Solving these equations yields

$$
\begin{aligned}
e_{u1}^{k+1} &= \frac{p_{i1}^k}{p_{i2}^k} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \qquad (A.13) \\
e_{u2}^{k+1} &= 0, \\
e_{u3}^{k+1} &= \frac{p_{i3}^k}{p_{i1}^k} e_{u1}^{k+1}, \\
& \ldots
\end{aligned}
$$

The equilibrium is a solution to $2(K+3)(K+1)$ price coefficients $p^k, \mathbf{p}_i^k$, and $\mathbf{p}_u^k$. Note that $p_{u2}^k=0$. Equations (A.4) and (A.10) can be combined to yield $K+1$ equations that can be solved for p^k, as will be done below. Combining equations (A.5) and (A.11) yields $(K+1)\times(K+3)$ equations and combining equations (A.6) and (A.13) yields the remaining $(K+1)\times(K+2)$ equations. Note that the equations for e_{u2}^k are redundant. When solving these equations, I substitute for the values of the conditional variances and covariances of returns computed above and that depend on the filtering problem of uninformed investors. A solution to this nonlinear system of equations constitutes a stationary rational expectations equilibrium.

It is possible to further characterize the equilibrium solution. Note that $e_{il}^{k+1}+e_{ul}^{k+1}=0$ for all l but for $l=2$. Thus, provided $t+1$ is not a dividend paying period (i.e., $K-1 \geq k \geq 0$), adding (A.5) and (A.6) gives

$$
\mathbf{e}_i^{k+1}+\mathbf{e}_u^{k+1}=\mathbf{p}_i^{k+1}\mathbf{A}_x-R\mathbf{p}_i^k+\mathbf{p}_u^{k+1}\mathbf{I}_{-2}\mathbf{A}_x-R\mathbf{p}_u^k\mathbf{I}_{-2},
$$

which can be simplified to yield

$$
\begin{aligned}
0 &= \left(p_{i1}^{k+1}+p_{u1}^{k+1}\right)\rho_F-R\left(p_{i1}^k+p_{u1}^k\right) \qquad (A.14) \\
e_{i2}^{k+1} &= p_{i2}^{k+1}\rho_Z-Rp_{i2}^k \qquad (A.15) \\
& \ldots \qquad (A.16) \\
0 &= p_{iK+3}^{k+1}+p_{uK+3}^{k+1}-R\left(p_{iK+2}^k+p_{uK+2}^k\right) \\
0 &= p_{iK+3}^k+p_{uK+3}^k.
\end{aligned}
$$

If $t+1$ is a dividend paying period (i.e., $K=k$), one obtains

$$
\mathbf{e}_i^0+\mathbf{e}_u^0=\left(\mathbf{p}_i^0+\mathbf{c}_{-2}\right)\mathbf{A}_x-R\mathbf{p}_i^K+\mathbf{p}_u^0\mathbf{I}_{-2}\mathbf{A}_x-R\mathbf{p}_u^K\mathbf{I}_{-2},
$$

which can be simplified to yield $0 = p^k_{u2}$, and

$$\begin{aligned} 0 &= \left(p^0_{i1} + 1 + p^0_{u1}\right)\rho_F - R\left(p^K_{i1} + p^K_{u1}\right) && \text{(A.17)} \\ e^0_{i2} &= p^0_{i2}\rho_Z - Rp^K_{i2} && \text{(A.18)} \\ &\ldots && \text{(A.19)} \\ 0 &= p^0_{iK+3} + 1 + p^0_{uK+3} - R\left(p^K_{iK+2} + p^K_{uK+2}\right) \\ 0 &= p^K_{iK+3} + p^K_{uK+3}. \end{aligned}$$

Putting it all together gives the following solution to the coefficients of the price function. First, I get the solution for $\{p^k\}$ from (A.4) and (A.10):

$$\begin{bmatrix} p^0 \\ p^1 \\ \ldots \\ p^K \end{bmatrix} = \frac{-1}{R^{K+1}-1} \begin{bmatrix} R^K & R^{K-1} & \ldots & 1 \\ 1 & R^K & \ldots & R \\ \ldots & \ldots & \ldots & \ldots \\ R^{K-1} & \ldots & 1 & R^K \end{bmatrix} \begin{bmatrix} e^1_0 \\ e^2_0 \\ \ldots \\ e^0_0 \end{bmatrix},$$

where each $p^k < 0$. Next, I get the solution for $\{p^k_{i2}\}$ by combining (A.12) with (A.15) and (A.18):

$$\begin{bmatrix} p^0_{i2} \\ \ldots \\ p^{K-1}_{i2} \\ p^K_{i2} \end{bmatrix} = \frac{-1}{R^{K+1}-\rho_F^{K+1}} \begin{bmatrix} \rho_F^K & R^K & R^{K-1}\rho_F & \ldots & R\rho_F^{K-1} \\ R\rho_F^{K-1} & \rho_F^K & R^K & \ldots & R^2\rho_F^{K-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ R^{K-1}\rho_F & R^{K-2}\rho_F^2 & R^{K-3}\rho_F^3 & \ldots & R^K \\ R^K & R^{K-1}\rho_F & R^{K-2}\rho_F^2 & \ldots & \rho_F^K \end{bmatrix} \begin{bmatrix} e^0_{i2} \\ \ldots \\ e^{K-1}_{i2} \\ e^K_{i2} \end{bmatrix}. \quad \text{(A.20)}$$

Note that if $Cov^i_{Qq,k} > 0$, for all k, then $p^k_{i2} < 0$, for all k.

Further, it is possible to solve for $p^k_{i1} + p^k_{u1}$ using (A.14) and (A.17):

$$p^k_{i1} + p^k_{u1} = \frac{R^k\rho_F^{K+1-k}}{R^{K+1}-\rho_F^{K+1}}.$$

Collecting the remaining equation in (A.16) and (A.19), I get $p^k_{iK+3} + p^k_{uK+3} = 0$ for all k, and

$$\begin{aligned} p^K_{iK+2} + p^K_{uK+2} &= R^{-1} \\ p^k_{iK+2} + p^k_{uK+2} &= 0, \qquad k = 0, \ldots, K-1 \end{aligned}$$

$$\begin{aligned} p^K_{iK+1} + p^K_{uK+1} &= R^{-1} \\ p^{K-1}_{iK+1} + p^{K-1}_{uK+1} &= R^{-2} \\ p^k_{iK+1} + p^k_{uK+1} &= 0, \qquad k = 0, \ldots, K-2 \end{aligned}$$

...

$$\begin{aligned}
p_{i3}^{K}+p_{u3}^{K} &= R^{-1} \\
p_{i3}^{K-1}+p_{u3}^{K-1} &= R^{-2} \\
&... \\
p_{i3}^{1}+p_{u3}^{1} &= R^{-K} \\
p_{i3}^{k}+p_{u3}^{k} &= 0, \qquad k=0.
\end{aligned}$$

To complete the proof, I now show that

$$\begin{aligned}
p_{iK+3}^{K} &= p_{uK+3}^{K}=0 \\
p_{iK+2}^{K-1} &= p_{uK+2}^{K-1}=p_{iK+3}^{K-1}=p_{uK+3}^{K-1}=0 \\
&... \\
p_{i4}^{1} &= p_{u4}^{1}=...=p_{iK+2}^{1}=p_{uK+2}^{1}=p_{iK+3}^{1}=p_{uK+3}^{1}=0 \\
p_{i3}^{0} &= p_{u3}^{0}=p_{i4}^{0}=p_{u4}^{0}=...=p_{iK+2}^{0}=p_{uK+2}^{0}=p_{iK+3}^{0}=p_{uK+3}^{0}=0.
\end{aligned}$$

I start with showing that $p_{iK+3}^{k}=p_{uK+3}^{k}=0$. Start with (A.6) and assume that $k+1\neq 0$, or $k<K$. It can be shown that

$$\mathbf{e}_u^{k+1} = \begin{bmatrix} p_{u1}^{k+1}\rho_F-Rp_{u1}^{k} \\ 0 \\ p_{u4}^{k+1}-Rp_{u3}^{k} \\ ... \\ p_{uK+3}^{k+1}-Rp_{uK+2}^{k} \\ -Rp_{uK+3}^{k} \end{bmatrix}^{\intercal} - \mathbf{p}_u^{k+1}\mathbf{K}_{k+1}^{k}\times \tag{A.21}$$

$$\begin{bmatrix} p_{i1}^{k+1}\rho_F-p_{i2}^{k+1}\frac{p_{i1}^{k}}{p_{i2}^{k}}\rho_Z & 0 & p_{i4}^{k+1}-p_{i2}^{k+1}\frac{p_{i3}^{k}}{p_{i2}^{k}}\rho_Z & ... & -p_{i2}^{k+1}\frac{p_{iK+3}^{k}}{p_{i2}^{k}}\rho_Z \\ \rho_F & 0 & ... & ... & 0 \\ 0 & 0 & 0 & ... & 0 \end{bmatrix}, \tag{A.22}$$

with the term on the right hand side post-multiplying $\mathbf{p}_u^{k+1}$ equal to

$$\begin{bmatrix} k_{11}^{k}\left(p_{i1}^{k+1}\rho_F-p_{i2}^{k+1}\frac{p_{i1}^{k}}{p_{i2}^{k}}\rho_Z\right)+k_{12}^{k}\rho_F & 0 & k_{11}^{k}\left(p_{i4}^{k+1}-p_{i2}^{k+1}\frac{p_{i3}^{k}}{p_{i2}^{k}}\rho_Z\right) & ... & -p_{i2}^{k+1}\frac{p_{iK+3}^{k}}{p_{i2}^{k}}\rho_Z k_{11}^{k} \\ k_{21}^{k}\left(p_{i1}^{k+1}\rho_F-p_{i2}^{k+1}\frac{p_{i1}^{k}}{p_{i2}^{k}}\rho_Z\right)+k_{22}^{k}\rho_F & 0 & k_{21}^{k}\left(p_{i4}^{k+1}-p_{i2}^{k+1}\frac{p_{i3}^{k}}{p_{i2}^{k}}\rho_Z\right) & & -p_{i2}^{k+1}\frac{p_{iK+3}^{k}}{p_{i2}^{k}}\rho_Z k_{21}^{k} \\ & ... & & & \\ & ... & & & \\ & 0 & & & -p_{i2}^{k+1}\frac{p_{iK+3}^{k}}{p_{i2}^{k}}\rho_Z k_{K+31}^{k} \end{bmatrix}.$$

Then, using the last row of (A.21) for any $k < K$, and letting $\mathbf{K}^k_{k+1,1.}$ denote the first column of $\mathbf{K}^k_{k+1}$,

$$e^{k+1}_{uK+3} = -Rp^k_{uK+3} + p^{k+1}_{i2}\frac{p^k_{iK+3}}{p^k_{i2}}\rho_Z \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}$$

Replacing the value of e^{k+1}_{uK+3} by its expression in (A.13) and using the fact that $p^k_{uK+3} + p^k_{iK+3} = 0$, the previous equality can be written as

$$\frac{p^k_{iK+3}}{p^k_{i2}}\frac{(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}{\lambda\left(\sigma^u_Q\right)^2+(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}\frac{Cov^i_t\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma^i_q\right)^2} = Rp^k_{iK+3}+p^{k+1}_{i2}\frac{p^k_{iK+3}}{p^k_{i2}}\rho_Z \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}.$$

Suppose $p^k_{iK+3} \neq 0$, then

$$\frac{1}{p^k_{i2}}\frac{(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}{\lambda\left(\sigma^u_Q\right)^2+(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}\frac{Cov^i_t\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma^i_q\right)^2} = R + \frac{p^{k+1}_{i2}}{p^k_{i2}}\rho_Z \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}. \tag{A.23}$$

Now note that for $k < K$ $(k+1 \neq 0)$, the next to last equation in (A.21) is

$$e^{k+1}_{uK+2} = p^{k+1}_{uK+3} - Rp^k_{uK+2} - \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}\left(p^{k+1}_{iK+3} - p^{k+1}_{i2}\frac{p^k_{iK+2}}{p^k_{i2}}\rho_Z\right),$$

or

$$\begin{aligned}&\frac{p^k_{iK+2}}{p^k_{i2}}\frac{(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}{\lambda\left(\sigma^u_Q\right)^2+(1-\lambda)\left(\sigma^i_Q\right)^2\left(1-\rho^2_{Qq}\right)}\frac{Cov^i_t\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma^i_q\right)^2} - Rp^k_{iK+2} \\ = \;& \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.} p^{k+1}_{i2}\frac{p^k_{iK+2}}{p^k_{i2}}\rho_Z - p^{k+1}_{iK+3}\left(1+\mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}\right).\end{aligned}$$

Again, replacing the value of e^{k+1}_{uK+2} and using (A.23), I arrive at

$$0 = -p^{k+1}_{iK+3}\left(1+\mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}\right).$$

Under the assumption that $p^{k+1}_{iK+3} \neq 0$, it must then be that $1 + \mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.} = 0$. But, from (A.15),

$$e^{k+1}_{i2} = p^{k+1}_{i2}\rho_Z - Rp^k_{i2},$$

which combined with (A.23) and the equilibrium value of e^{k+1}_{i2}, leads to

$$\frac{Cov^i_t\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma^i_q\right)^2} = p^{k+1}_{i2}\rho_Z\left(1+\mathbf{p}^{k+1}_u \mathbf{K}^k_{k+1,1.}\right).$$

This results in a contradiction unless the two assets are uncorrelated, i.e., $Cov_t^i(Q_{t+1}, q_{t+1}) = 0$. In this case however there is no private information and $\mathbf{p}_u = 0$, which is a contradiction. Next, I show that $p_{iK+2}^{K-1} = p_{uK+2}^{K-1} = 0$. (The proof for the other price coefficients is similar.) From the next to last equation in (A.21) written for $k = K - 1$:

$$\begin{aligned} p_{uK+3}^K - Rp_{uK+2}^{K-1} - \mathbf{p}_u^K \mathbf{K}_{K,1.}^{K-1} \left(p_{iK+3}^K - p_{i2}^K \frac{p_{iK+2}^{K-1}}{p_{i2}^{K-1}} \rho_Z \right) &= e_{uK+2}^K \\ -Rp_{uK+2}^{K-1} + \mathbf{p}_u^K \mathbf{K}_{K,1.}^{K-1} p_{i2}^K \frac{p_{iK+2}^{K-1}}{p_{i2}^{K-1}} \rho_Z &= e_{uK+2}^K. \end{aligned}$$

Using the expression for e_{uK+2}^K, $p_{iK+2}^{K-1} + p_{uK+2}^{K-1} = 0$, and letting $p_{iK+2}^{K-1} \neq 0$, then the last expression can be written as

$$R + \mathbf{p}_u^K \mathbf{K}_{K,1.}^{K-1} \frac{p_{i2}^K}{p_{i2}^{K-1}} \rho_Z = \frac{1}{p_{i2}^{K-1}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2 \left(1 - \rho_{Qq}^2\right)} \frac{Cov_t^i(Q_{t+1}, q_{t+1})}{\left(\sigma_q^i\right)^2},$$

which it was just shown to not hold. The contradiction implies $p_{iK+2}^{K-1} = p_{uK+2}^{K-1} = 0$.■

Proof of Proposition 3: Throughout, I shall use the result proved above that $e_{il}^{k+1} + e_{ul}^{k+1} = 0$ for all l but for $l = 2$. Using (10) and the expression for $E_t^u\left[Q_{t+1}^{k+1}\right]$, the stock demand of uninformed investors is

$$\theta_t^u = \frac{e_0^{k+1} + \left(\mathbf{e}_i^{k+1} + \mathbf{e}_u^{k+1}\right) \hat{\mathbf{x}}_t}{\gamma \left(\sigma_Q^u\right)^2} = \frac{e_0^{k+1} + e_{i2}^{k+1} \hat{Z}_t}{\gamma \left(\sigma_Q^u\right)^2}.$$

The asset demand of informed investors can be similarly obtained. Using (9) and the expression for $E_t^i\left[Q_{t+1}^{k+1}\right]$, I get

$$\begin{aligned} \theta_t^i &= \frac{e_0^{k+1} + e_{i2}^{k+1} Z_t - \mathbf{e}_u^{k+1}(\mathbf{x}_t - \hat{\mathbf{x}}_t)}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i Z_t}{\gamma \sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \\ &= \frac{e_0^{k+1}}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \left(\frac{\rho_{Qq}^i}{\gamma \sigma_Q^i \sigma_q^i \left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{e_{i2}^{k+1}}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \right) Z_t \\ &\quad - \frac{e_{u1}^{k+1}}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \left(F_t - \hat{F}_t\right) - \sum_{j=0}^{k-1} \frac{e_{uj+3}^{k+1}}{\gamma \left(\sigma_Q^i\right)^2 \left(1 - \left(\rho_{Qq}^i\right)^2\right)} \left(\varepsilon_{t-j}^D - \hat{\varepsilon}_{t-j}^D\right). \end{aligned}$$

Using the expression for e_{i2}^{k+1} in equation (A.12), the coefficient associated with Z_t can be written as

$$-\frac{\rho_{Qq}^{i}}{\gamma\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)},$$

which is negative if and only if $Cov_t^i\left(Q_{t+1},q_{t+1}\right)>0$. Summarizing, because $e_0^{k+1}>0$, $f_{i0}^{k+1}, f_{u0}^{k+1}>0$. Also, $f_{i1}^{k+1}<0$ and $f_{u1}^{k+1}>0$ if and only if $Cov_t^i\left(Q_{t+1},q_{t+1}\right)>0$. ■

Conditional volatility of returns and mean trading volume calculations: Compute the value of the stock return variance conditional only on knowing that returns are drawn from a period of type k:

$$\begin{aligned}
\sigma_{Q,k}^2 &= E_k\left[\left(Q_{t+1}^{k+1}-e_0^{k+1}\right)^2\right]\\
&= E_k\left[\left(\mathbf{e}_i^{k+1}\mathbf{x}_t+\mathbf{e}_u^{k+1}\hat{\mathbf{x}}_t+\mathbf{b}_Q^{k+1}\boldsymbol{\varepsilon}_{t+1}^{k+1}\right)^2\right]\\
&= E_k\left[E_t^u\left(\mathbf{e}_i^{k+1}\left(\mathbf{x}_t-\hat{\mathbf{x}}_t\right)+e_{i2}^{k+1}\hat{Z}_t+\mathbf{b}_Q^{k+1}\boldsymbol{\varepsilon}_{t+1}^{k+1}\right)^2\right]\\
&= \mathbf{b}_Q^{k+1}\Sigma_{\varepsilon\varepsilon}^{k+1}\left(\mathbf{b}_Q^{k+1}\right)^{\intercal}+\left(e_{i2}^{k+1}\right)^2E_k\left[\hat{Z}_t^2\right]+\mathbf{e}_i^{k+1}\Omega^k\left(\mathbf{e}_i^{k+1}\right)^{\intercal},
\end{aligned}$$

recalling that $E_k\left[\left(\mathbf{x}_t-\hat{\mathbf{x}}_t\right)\hat{Z}_t\right]=E_k\left[E_t^u\left(\left(\mathbf{x}_t-\hat{\mathbf{x}}_t\right)\hat{Z}_t\right)\right]=E_k\left[E_t^u\left(\mathbf{x}_t-\hat{\mathbf{x}}_t\right)\hat{Z}_t\right]=0$, using the law of iterated expectations. To compute $E_k\left[\hat{Z}_t^2\right]$, note that, by definition of Ω^k,

$$\hat{\mathbf{x}}_t\hat{\mathbf{x}}_t^{\intercal}=E_t^u\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]-\Omega^k.$$

Thus, $E_k\left[\hat{\mathbf{x}}_t\hat{\mathbf{x}}_t^{\intercal}\right]=E_k\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]-\Omega^k$. Turn now to the value of $E_k\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]$. First, note that conditional on (t,k), then $t-1$ is a $k-1$-period and, abusing notation slightly, $E_k\left[\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\intercal}\right]=E_{k-1}\left[\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\intercal}\right]$. Then,

$$\begin{aligned}
E_k\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right] &= E_k\left[E_{t-1}^i\left(\mathbf{x}_t\mathbf{x}_t^{\intercal}\right)\right]\\
&= \mathbf{A}_xE_{k-1}\left[\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\intercal}\right]\mathbf{A}_x^{\intercal}+\mathbf{B}_x\Sigma_{\varepsilon\varepsilon}^k\mathbf{B}_x^{\intercal}
\end{aligned}$$

which can be found by solving the system of $K+1$ equations on $E_k\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]$ and noting that

$$E_0\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]=\mathbf{A}_xE_K\left[\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\intercal}\right]\mathbf{A}_x^{\intercal}+\mathbf{B}_x\Sigma_{\varepsilon\varepsilon}^0\mathbf{B}_x^{\intercal}.$$

The value of $E_k\left[\hat{Z}_t^2\right]$ is the $(2,2)$ element of $E_k\left[\mathbf{x}_t\mathbf{x}_t^{\intercal}\right]-\Omega^k$.

To compute the mean trading volume, I solve for $\sigma^2_{\Delta\theta^u_t,k}$,

$$\begin{aligned}
\sigma^2_{\Delta\theta^u_t,k} &= E_k\left[\left(\theta^u_t - \theta^u_{t-1} - E_k\left(\theta^u_t - \theta^u_{t-1}\right)\right)^2\right] \\
&= E_k\left[\left(f^{k+1}_{u1}\hat{Z}_t - f^k_{u1}\hat{Z}_{t-1}\right)^2\right] \\
&= \left(f^{k+1}_{u1}\right)^2 E_k\left[\hat{Z}^2_t\right] + \left(f^k_{u1}\right)^2 E_{k-1}\left(\hat{Z}^2_{t-1}\right) - 2f^{k+1}_{u1}f^k_{u1}E_k\left(\hat{Z}_t\hat{Z}_{t-1}\right) \\
&= \left(f^{k+1}_{u1}\right)^2 E_k\left[\hat{Z}^2_t\right] + \left(f^k_{u1} - 2f^{k+1}_{u1}\rho_Z\right) f^k_{u1}E_{k-1}\left(\hat{Z}^2_{t-1}\right),
\end{aligned}$$

where the third equality uses the fact that conditioning on t being a k-type period then $t-1$ is a $k-1$-type period and $E_k\left(\hat{Z}^2_{t-1}\right) = E_{k-1}\left(\hat{Z}^2_{t-1}\right)$. The last equality uses the fact that the conditional error in the expression for $\hat{Z}_t$ is independent of $\hat{Z}_{t-1}$. ■

Proof of Corollary 1: Using the definition of $f(Q)$, the unconditional mean stock return is

$$E\left(Q_{t+1}\right) = \frac{1}{K+1}\sum_{k=0}^{K} E_k\left(Q_{t+1}\right) = \frac{1}{K+1}\sum_{k=0}^{K} e^k_0.$$

The unconditional variance in stock returns is

$$\begin{aligned}
Var\left(Q_{t+1}\right) &= \frac{1}{K+1}\sum_{k=0}^{K}\int \left(Q - E\left(Q_{t+1}\right)\right)^2 \phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ \\
&= \frac{1}{K+1}\sum_{k=0}^{K}\int \left(Q - e^k_0 + e^k_0 - E\left(Q_{t+1}\right)\right)^2 \phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ \\
&= \frac{1}{K+1}\sum_{k=0}^{K}\left(\sigma^2_{Q,k} + \left(e^k_0 - E\left(Q_{t+1}\right)\right)^2\right).
\end{aligned}$$

Finally, unconditional skewness is

$$\begin{aligned}
E\left[\left(Q - E\left(Q_{t+1}\right)\right)^3\right] &= \frac{1}{K+1}\sum_{k=0}^{K}\int \left(Q - E\left(Q_{t+1}\right)\right)^3 \phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ \\
&= \frac{1}{K+1}\sum_{k=0}^{K}\int \left(Q - e^k_0 + e^k_0 - E\left(Q_{t+1}\right)\right)^3 \phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ \\
&= \frac{1}{K+1}\sum_{k=0}^{K}\left[\left(e^k_0 - E\left(Q_{t+1}\right)\right)^3 + 3\sigma^2_{Q,k}\left(e^k_0 - E\left(Q_{t+1}\right)\right)\right]
\end{aligned} \tag{A.24}$$

The third equality uses $\int \left(Q - e^k_0\right)\phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ$ and the fact that skewness is zero for a normal variable, $\int \left(Q - e^k_0\right)^3 \phi\left(Q; e^k_0, \sigma^2_{Q,k}\right) dQ = 0$. The second term under the summation sign in (A.24) can be manipulated to yield the expression in the corollary by noting that

$$e_0^k - E\left(Q_{t+1}\right) = \frac{1}{K+1} \sum_{j=0, j \neq k}^{K} \left(e_0^k - e_0^j\right),$$

and grouping terms together under the last summation sign.■

The model without asymmetric information Guess the following price function

$$P_t^k = p^k + \frac{R^k \rho_F^{K+1-k}}{R^{K+1} - \rho_F^{K+1}} F_t + R^{-(K+1-k)} \sum_{j=0}^{k-1} \varepsilon_{t-j}^D + p_{i2}^k Z_t.$$

For simplicity assume $K = 2$. Using the guess for the price function, compute stock excess returns:

$$\begin{aligned} Q_t^0 &= P_t^0 + D_t^0 - R P_{t-1}^2 \\ &= p^0 - Rp^2 + \left(p_{i2}^0 \rho_Z - R p_{i2}^2\right) Z_{t-1} + p_{i2}^0 \varepsilon_t^Z + \frac{R^3}{R^3 - \rho_F^3} \varepsilon_t^F + \varepsilon_t^D, \end{aligned}$$

$$\begin{aligned} Q_t^2 &= P_t^2 - R P_{t-1}^1 \\ &= p^2 - Rp^1 + \left(p_{i2}^2 \rho_Z - R p_{i2}^1\right) Z_{t-1} + \frac{R^2 \rho_F}{R^3 - \rho_F^3} \varepsilon_t^F + R^{-1} \varepsilon_t^D + p_{i2}^2 \varepsilon_t^Z, \end{aligned}$$

and

$$\begin{aligned} Q_t^1 &= P_t^1 - R P_{t-1}^0 \\ &= p^1 - Rp^0 + \left(p_{i2}^1 \rho_Z - R p_{i2}^0\right) Z_{t-1} + \frac{R \rho_F^2}{R^3 - \rho_F^3} \varepsilon_t^F + R^{-2} \varepsilon_t^D + p_{i2}^1 \varepsilon_t^Z. \end{aligned}$$

Therefore, conditional expected excess returns are:

$$\begin{aligned} E_t\left[Q_{t+1}^0\right] &= p^0 - Rp^2 + \left(p_{i2}^0 \rho_Z - R p_{i2}^2\right) Z_t, \\ E_t\left[Q_{t+1}^2\right] &= p^2 - Rp^1 + \left(p_{i2}^2 \rho_Z - R p_{i2}^1\right) Z_t, \\ E_t\left[Q_{t+1}^1\right] &= p^1 - Rp^0 + \left(p_{i2}^1 \rho_Z - R p_{i2}^0\right) Z_t. \end{aligned}$$

I can now compute the conditional covariance with the private investment opportunity,

$$\begin{aligned} Cov_t^2\left(Q_{t+1}^0, q_{t+1}\right) &= \sigma_{Dq}, \\ Cov_t^1\left(Q_{t+1}^2, q_{t+1}\right) &= R^{-1} \sigma_{Dq}, \\ Cov_t^0\left(Q_{t+1}^1, q_{t+1}\right) &= R^{-2} \sigma_{Dq}, \end{aligned}$$

and the conditional return variance,

$$
\begin{aligned}
Var_t^2\left(Q_{t+1}^0\right) &= \left(\frac{R^3}{R^3-\rho_F^3}\right)^2\sigma_F^2+\sigma_D^2+\left(p_{i2}^0\right)^2\sigma_Z^2, \\
Var_t^1\left(Q_{t+1}^2\right) &= \left(\frac{R^2\rho_F}{R^3-\rho_F^3}\right)^2\sigma_F^2+R^{-2}\sigma_D^2+\left(p_{i2}^2\right)^2\sigma_Z^2, \\
Var_t^0\left(Q_{t+1}^1\right) &= \left(\frac{R\rho_F^2}{R^3-\rho_F^3}\right)^2\sigma_F^2+R^{-4}\sigma_D^2+\left(p_{i2}^1\right)^2\sigma_Z^2.
\end{aligned}
$$

Replacing these moments on the asset demands and imposing market clearing leads to a system of six equilibrium conditions in six unknowns $p^0, p^1, p^2, p_{i2}^0, p_{i2}^1, p_{i2}^2$:

$$
\begin{aligned}
\frac{1}{e_0^1}\left[p^1-Rp^0\right] &= 1 \\
\frac{1}{e_0^0}\left[p^0-Rp^2\right] &= 1 \\
\frac{1}{e_0^2}\left[p^2-Rp^1\right] &= 1 \\
\frac{1}{e_0^1}\left(p_{i2}^1\rho_Z-Rp_{i2}^0\right) &= \frac{\lambda\rho_{Qq,0}}{\gamma\sigma_{Q,0}\sigma_q\left(1-\left(\rho_{Qq,0}\right)^2\right)} \\
\frac{1}{e_0^0}\left(p_{i2}^0\rho_Z-Rp_{i2}^2\right) &= \frac{\lambda\rho_{Qq,2}}{\gamma\sigma_{Q,2}\sigma_q\left(1-\left(\rho_{Qq,2}\right)^2\right)} \\
\frac{1}{e_0^2}\left(p_{i2}^2\rho_Z-Rp_{i2}^1\right) &= \frac{\lambda\rho_{Qq,1}}{\gamma\sigma_{Q,1}\sigma_q\left(1-\left(\rho_{Qq,1}\right)^2\right)},
\end{aligned}
$$

where

$$
e_0^{k+1}\equiv E_k\left[Q_{t+1}^{k+1}\right]=\gamma\sigma_{Q,k}^2\frac{1-\rho_{Qq,k}^2}{1-(1-\lambda)\rho_{Qq,k}^2}.
$$

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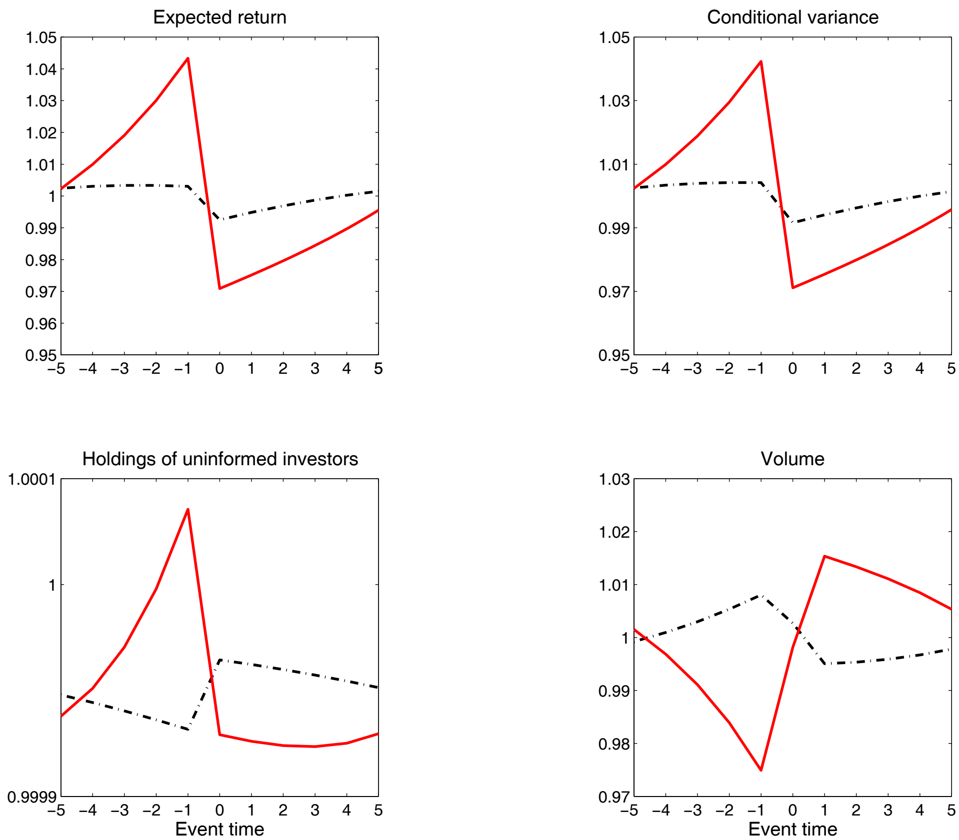

Figure 1: **Model without asymmetric information.** Pictures depict the event window around the dividend announcement, which occurs at date 0. Expected return is $E_k\left[Q_{t+1}^{k+1}\right]$, conditional volatility is $\sigma^2_{Q,k}$, holdings of uninformed investors is $(1-\lambda)\,E_k\left[\theta_t^u\right]$ and conditional trading volume is $E_k\left[Vol_t\right]$. Variables have been normalized to have mean of one. The solid line is for $\sigma_Z^2 = 2$ and the dashed line is for $\sigma_Z^2 = 5$. Public signals are fully informative, $\sigma^2_{Dk} = \sigma^2_{Fk} = 0$. Remaining parameters are: $K = 10$, $\sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

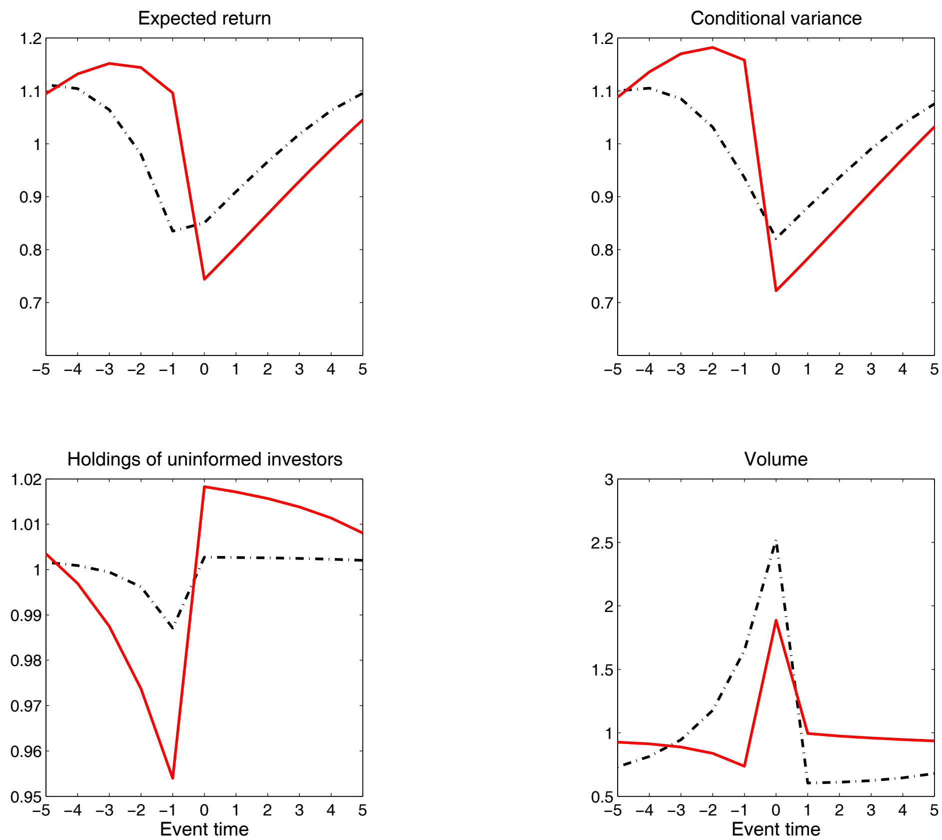

Figure 2: **Model with asymmetric information.** Pictures depict the event window around the dividend announcement, which occurs at date 0. Expected return is $E_k\left[Q_{t+1}^{k+1}\right]$, conditional volatility is $\sigma^2_{Q,k}$, holdings of uninformed investors is $(1-\lambda)\,E_k\left[\theta^u_t\right]$ and conditional trading volume is $E_k\left[Vol_t\right]$. Variables have been normalized to have mean of one. The solid line is for $\sigma^2_Z = 1$ and the dashed line is for $\sigma^2_Z = 5$. Public signals are not informative, $\sigma^2_{Dk}, \sigma^2_{Fk} \to \infty$. Remaining parameters are: $K = 10$, $\sigma^2_D = \sigma^2_q = \sigma^2_F = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

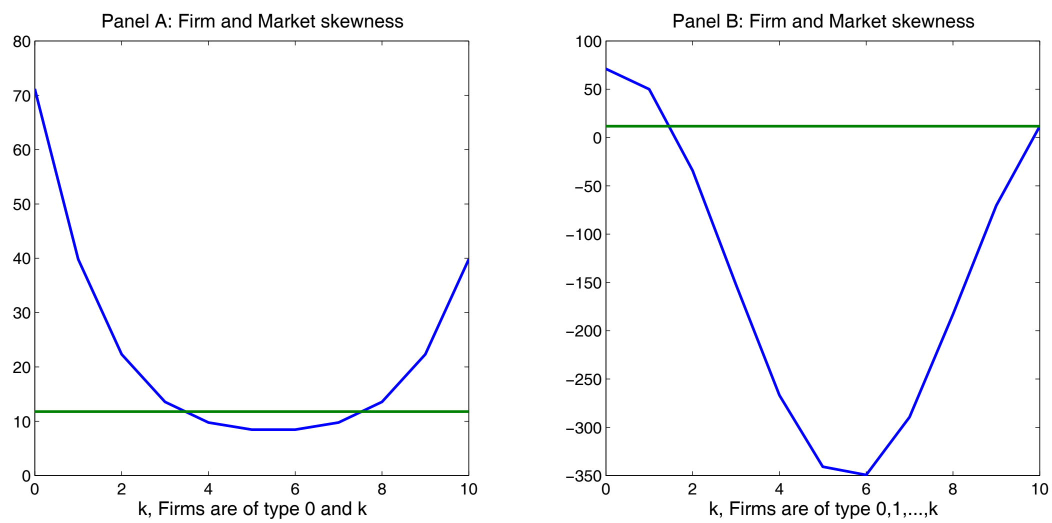

Figure 3: **Market skewness: comparative statics.** Panel A depicts market skewness when the market is composed of 2 types of firms with cash payout dates of 0 and k, respectively. Panel B depicts market skewness when the market is composed of k different types of firms with cash payout dates of $0, 1, ..., k$, respectively. The horizontal line above the origin depicts firm level skewness in each equilibrium. Parameters are: $K = 10$, $\sigma^2_{Dk} = \sigma^2_{Fk} = 0.5, \sigma^2_D = \sigma^2_q = \sigma^2_F = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

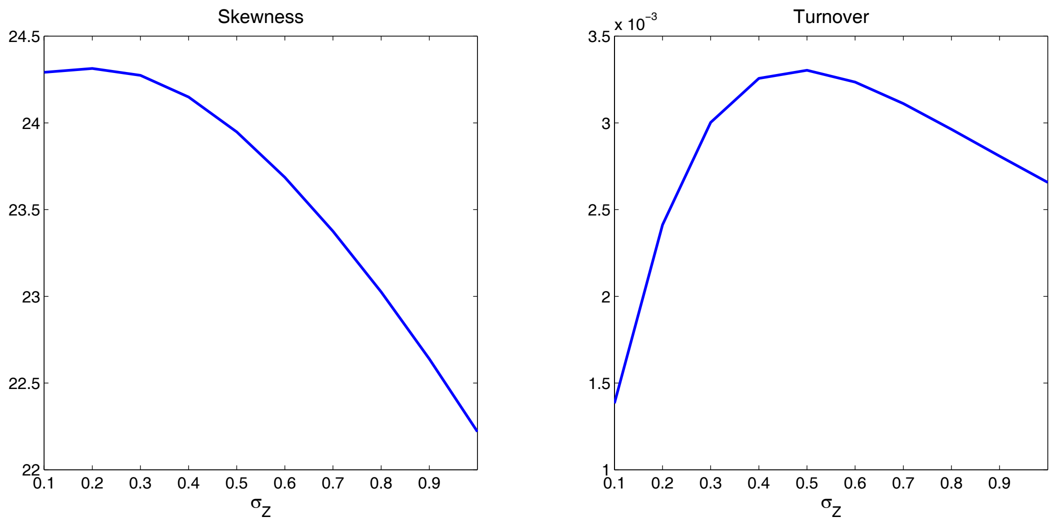

Figure 4: **Model without asymmetric information: The effect of liquidity shocks.** Public signals are fully informative, $\sigma^2_{Dk} = \sigma^2_{Fk} = 0$. The parameters used are: $K = 10$, $\sigma^2_D = \sigma^2_q = \sigma^2_F = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

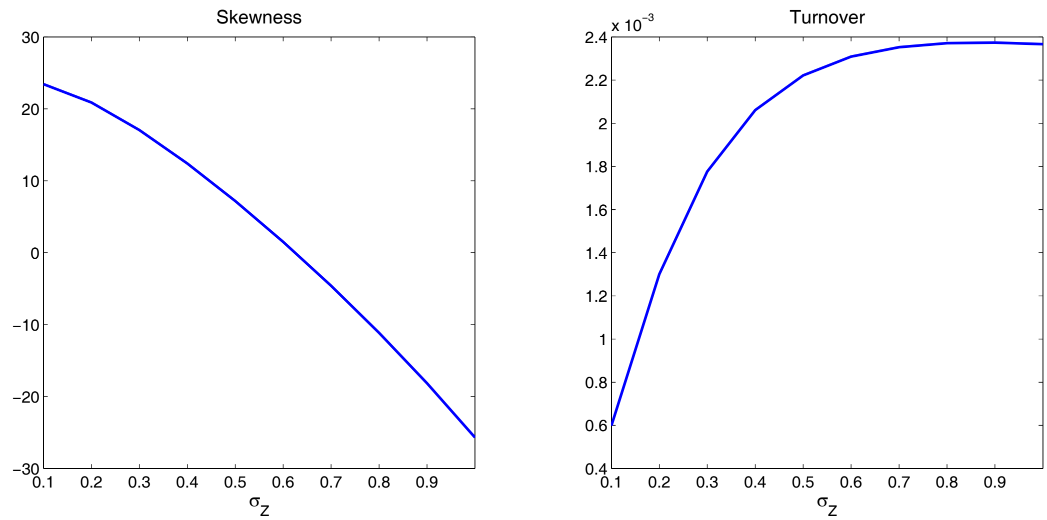

Figure 5: **Model with asymmetric information: The effect of liquidity shocks.** Public signals are not informative, $\sigma^2_{Dk}, \sigma^2_{Fk} \to \infty$. The parameters used are: $K = 10$, $\sigma^2_D = \sigma^2_q = \sigma^2_F = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

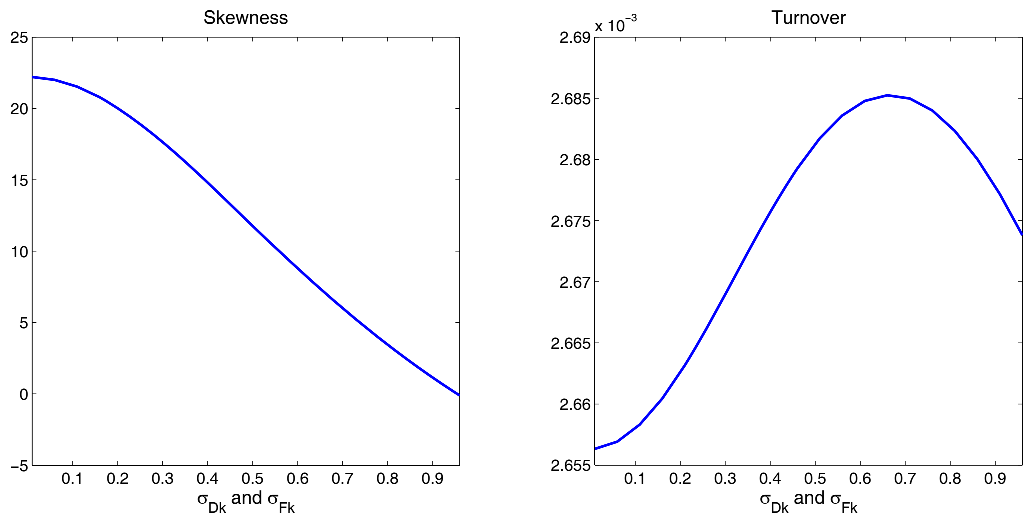

Figure 6: **Model with asymmetric information: The effect of information asymmetry**. The parameters used are: $K = 10$, $\sigma_Z^2 = \sigma_D^2 = \sigma_q^2 = \sigma_F^2 = 1$, $\sigma_{Dq} = .5$, $\rho_F = \rho_Z = .9$, $\gamma = 5$, $\lambda = .5$, $R = 1.0025$.

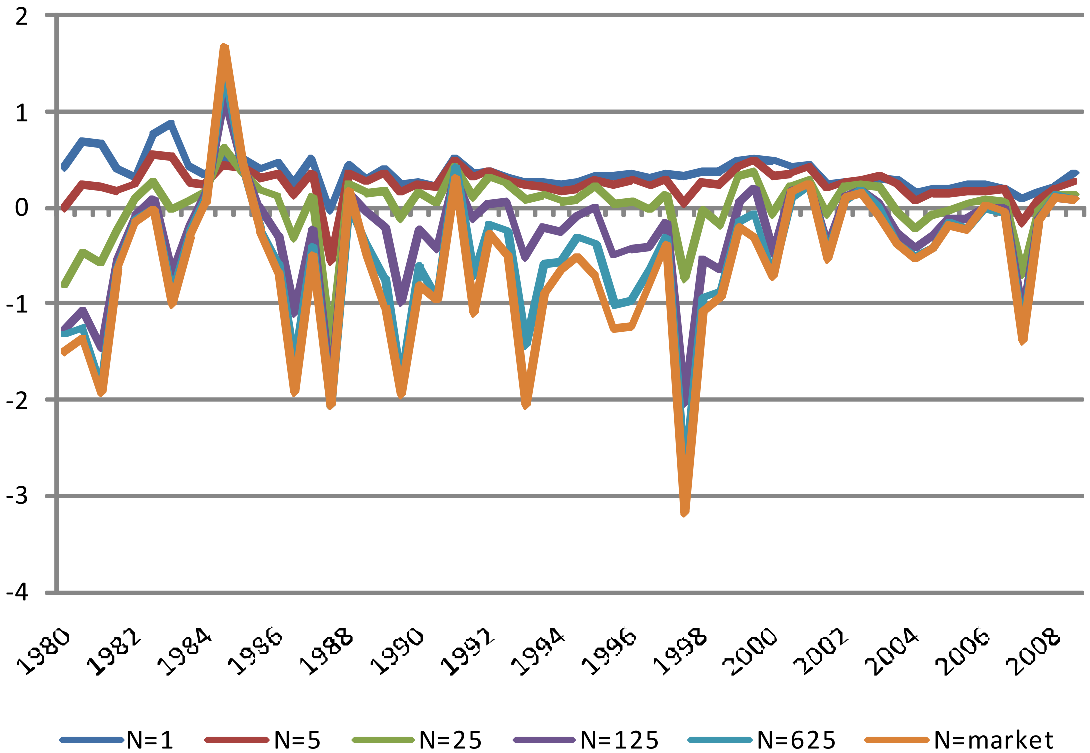

Figure 7: **Median skewness in daily returns across portfolios with N firms.** Skewness is computed using equally-weighted portfolios and six months of trading data. When $N = 1$, the figure gives firm-level skewness. The portfolios are constructed by randomly ranking the firms and then grouping them. If two firms are in the same portfolio when $N = 5$, then they will also be in the same portfolio for higher N. The larger portfolio, labelled "Market" comprises all of the firms in CRSP in the specific year and semester. Only firms for which trading data exists for all days in the semester are considered.

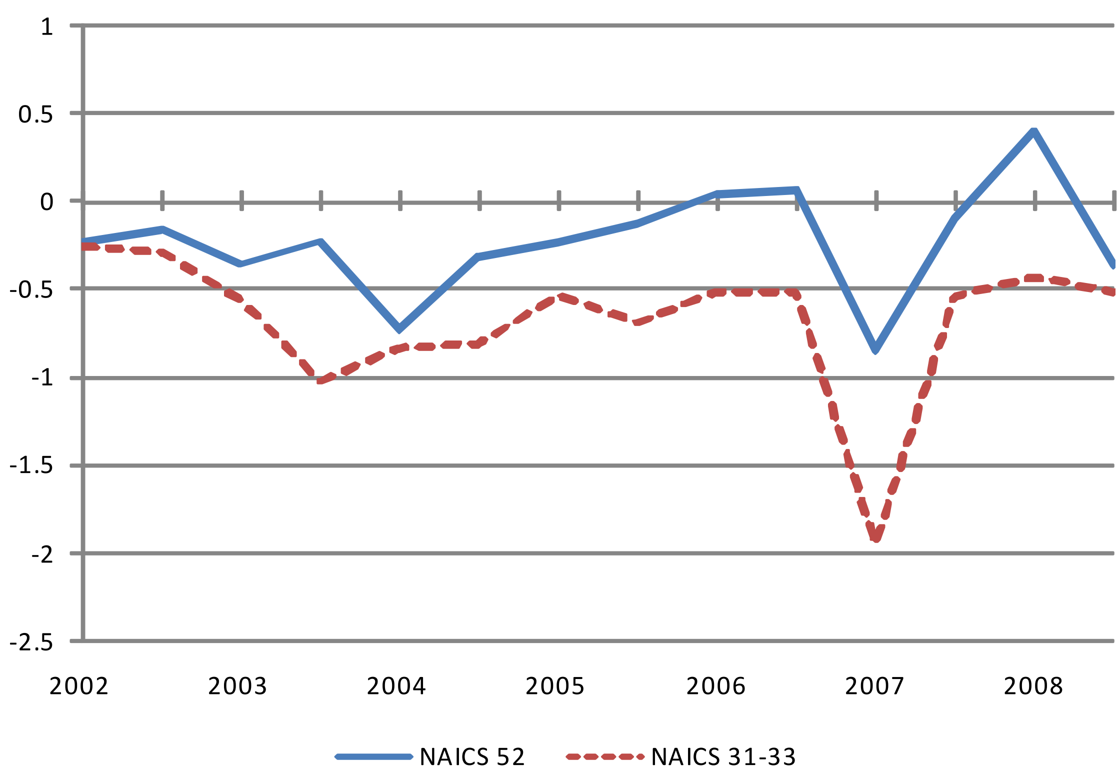

Figure 8: **Aggregate skewness minus firm-level skewness per industry.** NAICS 52 is Finance and Insurance and NAICS 31-33 is Manufacturing.

Table 1

Skewness in stock returns

The table presents the median skewness in daily returns from portfolios of size N. Skewness is computed using equally-weighted portfolios and 6-month of trading data. When N=1, the table gives firm-level skewness. The portfolios are constructed by randomly ranking the firms and then grouping them. If two firms are in the same portfolio when N=5, then they will also be in the same portfolio for higher N. The larger portfolio, labelled "Market" comprises all of the firms in CRSP in the specific year and semester. Only firms for which trading data exists for all days in the semester are considered.

		Median Skewness in portfolios with N firms						Trading	Number
Year	Semester	1	5	25	125	625	Market	days	of firms
1980	1	0.408	-0.010	-0.801	-1.274	-1.313	-1.516	126	4398
	2	0.680	0.253	-0.472	-1.077	-1.266	-1.381	127	4334
1981	1	0.672	0.226	-0.584	-1.471	-1.833	-1.935	125	4511
	2	0.401	0.190	-0.253	-0.544	-0.628	-0.665	128	4674
1982	1	0.310	0.255	0.089	-0.093	-0.132	-0.165	125	4646
	2	0.759	0.562	0.271	0.077	-0.021	-0.041	128	4762
1983	1	0.863	0.549	-0.015	-0.697	-0.919	-1.022	126	5112
	2	0.406	0.274	0.069	-0.195	-0.266	-0.318	127	5328
1984	1	0.336	0.251	0.156	0.116	0.084	0.025	126	5744
	2	0.494	0.454	0.625	1.147	1.449	1.661	127	5703
1985	1	0.508	0.424	0.371	0.435	0.439	0.459	125	5744
	2	0.400	0.314	0.187	-0.017	-0.235	-0.260	127	5657
1986	1	0.466	0.353	0.122	-0.296	-0.589	-0.676	125	5826
	2	0.228	0.128	-0.302	-1.113	-1.639	-1.928	128	5921
1987	1	0.503	0.348	0.110	-0.241	-0.417	-0.517	125	6317
	2	-0.034	-0.566	-1.451	-1.932	-2.053	-2.062	128	6557
1988	1	0.443	0.367	0.238	0.150	0.038	0.171	126	6676
	2	0.291	0.274	0.154	-0.049	-0.381	-0.519	127	6548
1989	1	0.403	0.362	0.173	-0.215	-0.740	-1.034	126	6523
	2	0.238	0.146	-0.119	-1.001	-1.839	-1.963	126	6408
1990	1	0.263	0.248	0.150	-0.228	-0.601	-0.820	126	6436
	2	0.220	0.234	0.047	-0.431	-0.944	-0.971	127	6424
1991	1	0.494	0.490	0.414	0.343	0.415	0.297	125	6380
	2	0.346	0.324	0.122	-0.135	-0.724	-1.110	128	6364
1992	1	0.379	0.378	0.305	0.034	-0.190	-0.295	126	6382
	2	0.309	0.291	0.248	0.049	-0.242	-0.510	128	6483
1993	1	0.266	0.246	0.087	-0.516	-1.442	-2.062	125	6681
	2	0.268	0.225	0.124	-0.208	-0.583	-0.938	128	6949
1994	1	0.236	0.172	0.068	-0.265	-0.571	-0.674	125	7449
	2	0.260	0.203	0.100	-0.094	-0.327	-0.527	127	7709
1995	1	0.321	0.289	0.217	-0.017	-0.377	-0.725	126	7764
	2	0.316	0.241	0.036	-0.496	-1.014	-1.278	126	7843
1996	1	0.357	0.292	0.061	-0.427	-0.973	-1.237	126	8067
	2	0.305	0.213	-0.022	-0.408	-0.651	-0.809	128	8411
1997	1	0.339	0.288	0.132	-0.161	-0.309	-0.396	125	8624
	2	0.320	0.045	-0.741	-2.043	-2.821	-3.190	128	8564
1998	1	0.376	0.267	-0.015	-0.538	-0.952	-1.111	124	8572
	2	0.362	0.253	-0.182	-0.664	-0.877	-0.957	128	8383
1999	1	0.492	0.437	0.340	0.031	-0.168	-0.226	124	8094
	2	0.506	0.495	0.377	0.194	-0.081	-0.322	128	7871
2000	1	0.481	0.329	-0.078	-0.484	-0.668	-0.730	126	7856
	2	0.412	0.348	0.221	0.155	0.075	0.140	126	7723
2001	1	0.439	0.419	0.299	0.217	0.228	0.227	125	7522
	2	0.233	0.205	-0.069	-0.412	-0.483	-0.549	123	7212
2002	1	0.266	0.260	0.219	0.100	0.074	0.103	124	7029
	2	0.264	0.300	0.247	0.159	0.166	0.131	128	6809
2003	1	0.304	0.342	0.218	0.027	-0.001	-0.107	124	6637
	2	0.287	0.247	-0.044	-0.282	-0.376	-0.402	128	6280
2004	1	0.150	0.069	-0.213	-0.443	-0.507	-0.546	124	6420
	2	0.195	0.145	-0.059	-0.306	-0.420	-0.438	128	6428
2005	1	0.184	0.153	-0.019	-0.119	-0.158	-0.197	125	6468
	2	0.229	0.170	0.034	-0.122	-0.230	-0.235	127	6446
2006	1	0.246	0.171	0.084	-0.007	-0.006	-0.004	125	6471
	2	0.198	0.196	0.067	-0.022	-0.041	-0.060	126	6519
2007	1	0.079	-0.149	-0.702	-1.154	-1.297	-1.400	124	6495
	2	0.153	0.065	-0.051	-0.148	-0.155	-0.163	127	6588
2008	1	0.221	0.200	0.142	0.090	0.141	0.089	125	6640
	2	0.337	0.277	0.129	0.079	0.087	0.080	128	6609